Rising Wealth Inequality: Intergenerational Links, Entrepreneurship, and the Decline in Interest Rate *

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Abstract

The share of wealth held by the top one percent of Americans has increased from about 24% in 1980 to 40% in 2010. This paper examines the potential role played by three factors in accounting for this increase - decline in the corporate tax rates, increase in the income risk, and the decline in the world interest rates. Our model consists of altruistic households who either run a business or work for others. Entrepreneurial households enjoy high returns due to high productivity while worker households' savings earn the bank deposit rate that is determined in a competitive banking sector and equals the rate of return on foreign bonds. We find that entrepreneurship and intergenerational links via altruism are important factors generating a wealth distribution that mimics the data in the 2000s. However, it is the decline in the interest rate that plays a major role in accounting for the increase in the U.S. wealth inequality since the 1980s. In our model, the decline in the interest rate increases wealth inequality as it affects the two types of households differently. Entrepreneurial households benefit from lower financing costs and increase their investments while worker households face lower returns to their savings as the interest rate declines. Other changes such as the changes in taxation and income risk play a less significant role.

Keywords: inequality, intergenerational links, saving, lifecycle models

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1 Introduction

It is well known that wealth inequality in the United States is high and has been rising over the past decades. For instance, the share of wealth held by the top one percent in the U.S. has increased from 24% in 1980 to 40% in 2010 (see Saez and Zucman, 2016). In this paper, we develop a quantitative general equilibrium model to examine the potential driving forces behind the rise in wealth inequality in the U.S. since 1980, including the changes in taxation, the increase in income risk, and the decline in the world interest rates. We are particularly interested in the potential role played by the decline in interest rate that may have been caused by the large capital inflows into the U.S. from emerging economies such as Asia and oil exporting economies over the last few decades. In a model with entrepreneurship, declining interest rates affects different types of households very differently. Entrepreneurial households benefit from lower financing costs and increase their investments while worker households face lower returns to their savings as the interest rate declines. We find that the decline in the interest rate does play a significant role in accounting for the increase in wealth inequality.

Our model consists of altruistic households who choose to either run a private business, that is, become an entrepreneur, or work as an employee of other firms. Entrepreneurs face credit constraints and can finance investment only by using the family's own capital together with a limited amount of external funds from the banking sector. Entrepreneurs operate their firms and employ workers that belong to other families. Firm-specific productivity shocks determine the returns from running a business. Worker households face idiosyncratic labor income risk and can only deposit their savings in the banking sector, earning a deposit rate that is determined in a competitive banking sector.

As in İmrohoroğlu and Zhao (2018, 2020), individuals drive utility from their own lifetime consumption and from the felicity of their predecessors and descendants. Since parents care about the utility of their descendants, they save to insure them against the labor income risk, and since children are altruistic toward their parents, they support them during retirement and insure them against mortality risks. In addition, parents leave voluntary bequests to their children. We find that these intergenerational links play an important role in the model-generated wealth inequality. The key difference between the two types of households is that worker households put their savings in a bank account whereas entrepreneurial households invest all their savings in their entrepreneurial firms. A standard production sector with corporations paying the corporate profits tax exists in the background. Workers pay labor income taxes and entrepreneurial firms pay taxes on their profits.

We calibrate this model so that the steady state of the model economy resembles the U.S. economy in the 2000s in terms of its wealth distribution, having taken the tax system and the world interest rates as given. We then solve the time series path of the model economy from 1980 towards the present including the changes in tax rates, income risk, and world interest rates observed in the U.S. during this time period. We treat the U.S. as an open economy where the time path of the interest rate is dictated by the world events. Using a series of counterfactual experiments, we examine the contribution of each exogenous change to the rise in wealth inequality since the 1980s.

We find that the decline in the interest rate plays a major role in accounting for the increase in wealth inequality. In the benchmark economy, the share of total wealth held by the top one percent increases by 7.8 percentage points from 1980 to 2010. Most of this result is due to the decline in the interest rate during this period. Other changes such as the decline in the corporate tax rate play a less significant role. Interestingly, we find that the increase in labor income risks (as documented in Heathcote, Storesletten, and Violante, 2010), an important dimension of the increasing income inequality, reduces wealth inequality. This is because rising labor income risk increases precautionary saving by worker households, and this impact tends to be more profound among households on the bottom tail of the distribution.

It is interesting to note that while the decline in interest rate substantially increases wealth inequality, it is welfare-improving in our model. Our quantitative experiment suggests that the newborn households living through the transition path featuring the decline in interest rate would, on average, enjoy a 3.2% welfare gain (measured in terms of consumption equivalent variation (CEV)), compared to those living through the counterfactual transition path with a constant interest rate. In addition, the welfare gain is larger for worker households than entrepreneurs, 3.3% v.s. 1.6%. These welfare results are largely due to the general equilibrium effects. The decline in interest rate raises the equilibrium wage rate and results in a decline in the variance of consumption. Both of these changes benefit workers more than the entrepreneurs.

Different papers have reached different conclusions regarding the main drivers of the rise in wealth inequality in the U.S. While Hubmer, Krusell, and Smith (2018) find the decrease in tax progressivity the most important driver of the rise in wealth inequality since the 1960s, Kaymak and Poschke (2016) conclude that the main driver is the increase in income inequality during the same period. One of the reasons behind these different findings seems to rest on the success of the model in matching the right wealth distribution in the data. Kaymak and Poschke (2016) rely on an extreme earnings inequality assumption to generate wealth inequality that mimics the data. In Hubmer, Krusell, and Smith (2018), stochastic returns to saving and return heterogeneity that are calibrated to micro data help deliver the right distribution of wealth. Indeed, they conclude that return heterogeneity is central to an understanding of wealth inequality.[1]

In our model, returns to wealth are heterogeneous since entrepreneurs can receive a higher return to capital than what is available as the deposit rate through the banks to worker households. In addition, altruistic preferences lead to differences in saving rates across household types where the income of the parent and the children differ from each other. Thus, our model generates a positive relation between lifetime income levels and saving rates, a property that has been difficult to capture in standard life-cycle models. Our model also incorporates several features that have been discussed to be important in understanding the distribution of wealth in the U.S. such as skewed earnings, differential saving rates across wealth levels, and stochastic idiosyncratic returns to wealth as in Benhabib, Bisin, and Luo (2019) and high returns earned by

[1] Some other explanations for the increase in wealth inequality include uneven wage and house price growth across cities (Greaney, 2020), global integration and foreign capital pressing up domestic asset prices held mostly by affluent households (Kim, 2020), and an increase in banks' willingness to fund risky entrepreneurial projects and a rise in the costs of starting a business (Mohaghegh, 2019).

entrepreneurs as in Quadrini (2000) and Cagetti and De Nardi (2006).[2]

Our findings indicate that intergenerational links as in İmrohoroğlu and Zhao (2018, 2020) together with entrepreneurship as in Quadrini (2000) and Cagetti and De Nardi (2006) are capable of generating a highly unequal wealth distribution as in the U.S. data. We find that a decline in interest rate in such a model framework is able to generate a substantial increase in U.S. wealth inequality since 1980 while changes in taxation and income inequality play a less important role in shaping the time path of U.S. wealth inequality during the same period.[3]

The remainder of the paper is organized as follows. Section 2 presents the model used in the paper and Section 3 its calibration. The main quantitative findings are presented in Section 4. Section 5 presents the results of some further analysis, and Section 6 provides the concluding remarks.

2 The Model

2.1 Households

The economy is populated with overlapping generations of individuals who enter the economy as an adult at age 1 and can live up to age $2T$. Individuals retire at age $T+1$ when their adult children are at age 1, and after retirement they face random lives. In this paper, we abstract from population growth and assume that the fertility rate is equal to the replacement rate. Consequently, a household consists of a child and a parent (if alive). Depending on survival, an individual's life overlaps with the parent's life in the first T periods and with the life of his/her child in the last T periods. A household lasts T periods. A dynasty is a sequence of households that belong to the same family line. At age $T+1$, the child becomes a parent in the next-generation household of the dynasty.

As in İmrohoroğlu and Zhao (2018, 2020), there is two-sided altruism where individuals derive utility from their own lifetime consumption and from the felicity of their predecessors and descendants.[4] At birth, an individual receives the realization of a random variable $z \in Z = \{H, L\}$ that determines lifetime labor ability where z is a two-state, first-order Markov process with the transition probability matrix $\Omega_z(z, z')$ where z' is the labor ability of the child in the dynasty and z is the labor ability of the parent. If $z = H$, an individual has a higher labor productivity throughout his/her lifespan than an individual with $z = L$.

Each period, every working-age individual gets to choose to either run a private business, that is, become an entrepreneur, or work as an employee, either for an entrepreneurial firm or in the corporate sector. Labor is supplied inelastically once the occupational choice is made. After retirement, individuals face an uncertain

[2]Some of the other papers focusing on wealth inequality in the U.S. include Castaneda Diaz-Gimenez, and Rios-Rull (2003); De Nardi (2004); Benhabib, Bisin, and Zhu (2011); Benhabib, Bisin, and Luo (2017); Gabaix et al. (2016); Saez and Zucman (2016); De Nardi and Yang (2016); and Cooke, Lee, and Zhao (2017).

[3]It is important to note that the time period we focus on is more recent than that studied in Kaymak and Poschke (2016) and Hubmer, Krusell, and Smith (2018), and thus our findings are not necessarily against what they found. Indeed, according to Piketty and Saez (2007) and Heathcote et al. (2010), the U.S. experienced much more changes in taxation and income risks in the 1960s and 1970s than in the 1980s and onward.

[4]This modeling strategy is similar to Laitner (1992), and Fuster, İmrohoroğlu, and İmrohoroğlu (2003, 2007).

lifespan where $d = 1$ indicates that the retired parent is alive and $d = 0$ indicates otherwise. If alive, a retired parent receives social security income, $SS(z_f)$, which is a function of lifetime labor ability, z_f. When the new household forms at time $T+1$, the adult child in the household receives the remaining assets (i.e., bequests).

Worker Households

The labor income of the adult child with a lifetime ability, z_s, who chooses to be a worker consists of an economy-wide wage rate w, an age-specific deterministic component, ε_j, and a stochastic component, μ_j. The stochastic component of labor income are governed by a first order Markov Chain with the transition probability matrix $\Omega(\mu, \mu')$. The total family income of these worker households, e_j, is given by the labor income of the age-j adult child and the social security income of the parent:

$$e_j = w\varepsilon_j\mu_j z_s + dSS(z_f) - \tau(w\varepsilon_j\mu_j z_s) \tag{1}$$

where $\tau(.)$ represents the labor income tax function.

We assume that worker households can only deposit their savings, a_j, in the banking sector earning a deposit rate r, which is determined in a competitive banking sector. The budget constraint facing worker households is then given by:

$$a_{j+1} + c_{sj} + dc_{fj} = e_j + (1+r)a_j + \kappa \tag{2}$$

where c_{sj} and c_{fj} are consumption of the child and the parent, and κ is a government transfer, which will be discussed later.

Entrepreneurial Households

An adult child who chooses to be an entrepreneur faces a credit constraint and can finance investment only by the family's own capital together with a limited amount of external funds from the banking sector. Following the literature, we model the credit constraint as an incentive-compatibility constraint where the firm can only pledge to repay an amount that is equivalent to a share η of its own capital.[5] The entrepreneur operates the firm and employs workers that belong to the other families.

The optimization problem of the entrepreneurial firm with own capital, a, is simply to choose the amount of loans, l, and number of workers, n_e, to maximize profits subject to the credit constraint given by:

$$l \leq \eta a \tag{3}$$

where η is the parameter governing the tightness of the credit constraint. We assume that entrepreneurial firms have access to a decreasing return to scale production technology with a sectoral level of productivity

[5]The same modeling strategy can be seen in Quadrini (2000), Cagetti and De Nardi (2006), for example.

A_e, and a firm-specific productivity shock, θ, which follows a first-order Markov process with the transition probability matrix $\Omega_\theta(\theta, \theta')$. The problem of an entrepreneurial firm is given by:

$$\pi(a, \theta) = \max_{l, n_e} A_e \theta \left[(a+l)^\alpha n_e^{1-\alpha} \right]^\phi - (\delta + r)(a + l) - wn_e \tag{4}$$

subject to equation 3. Here α is the output share of capital, and δ is the capital depreciation rate. The degree of decreasing returns from investment faced by the entrepreneurial firm is governed by ϕ. Profit maximization of the firm implies that the marginal product of capital is equal to $(\delta + r)$ if the credit constraint is not binding and it is larger than $(\delta + r)$ if otherwise. Given the optimal behavior of the firm, an entrepreneurial household faces the following budget constraint:

$$a_{j+1} + c_{sj} + dc_{fj} = dSS(z_f) + (1 + r)a_j + (\pi(a_j, \theta) - \tau_k(\pi)) + \kappa \tag{5}$$

where c_{sj} and c_{fj} are consumption of the child and the parent, a_j is the family's own assets, $\pi(a, \theta)$ is the maximized profit, and $\tau_k(\pi)$ is the tax imposed on business income from the entrepreneurial firm.

In this framework, since parents care about the utility of their descendants, they save to insure them against the labor income risk, and since children are altruistic toward their parents, they support them during retirement and insure them against mortality risks. In addition, parents leave voluntary bequests to their children. A key difference between the two types of households is that worker households put their saving in a bank account and entrepreneurial households invest all their saving in their firm.

Occupational Choice

The state of an age-j household (where j indicates the age of the adult child) is denoted by $x = (a, \mu, z_s, z_f, \theta, d)$. At the beginning of each period, the adult child faces the occupational choice between being a worker or an entrepreneur, which is given by:

$$V_j(x) = max \left\{ V_j^w(x), V_j^e(x) \right\}, \tag{6}$$

where V^w is the value function of being a worker household and V^e is the value function of being an entrepreneur household.

The problem of a worker household is to choose a sequence of consumption and asset holdings given the set of prices and policy parameters to maximize:

$$V_j^w(x) = \max_{c_s, c_f, a'} [u(c_s) + du(c_f)] + \beta E[\tilde{V}_{j+1}(x')] \tag{7}$$

subject to Equation (2), $a_j \geq 0, c_s \geq 0$ and $c_f \geq 0$, where

$$\tilde{V}_{j+1}(x') = \begin{cases} V_{j+1}(x') \text{ for } & j = 1, 2, ..., T-1 \\ V_1(x') \text{ for } & j = T. \end{cases}$$

The problem of an entrepreneurial household is to choose a sequence of consumption, asset holdings, and bank loans given the set of prices and policy parameters to maximize:

$$V_j^e(x) = \max_{c_s, c_f, a', l} [u(c_s) + du(c_f)] + \beta E[\tilde{V}_{j+1}(x')] \tag{8}$$

subject to Equation (5), Equation (3), $a_j \geq 0, c_s \geq 0$ and $c_f \geq 0$.

2.2 Standard Production Sector

Other than the entrepreneurial firms, the economy also contains a standard production sector featuring competitive firms that produce goods using a Cobb-Douglas production function $Y = AK^{\alpha}N^{1-\alpha}$ where K and N are the capital and labor input and A is the technology parameter that grows at the rate of g_A. Capital also depreciates at a constant rate δ.

The profit-maximizing behaviors of the competitive firms imply that the before tax rate of net return to capital used in the standard production sector, r_c, and the wage rate, w, are given by:

$$r_c = \alpha A(K/N)^{\alpha-1} - \delta \tag{9}$$

$$w = (1-\alpha)A(K/N)^{\alpha}. \tag{10}$$

There exists a corporate tax rate, τ_p, that is imposed on the firm's output net of wage cost and capital depreciation. That is, the firm's tax obligations are $\tau_p(Y - wN - \delta K)$.

2.3 Banks

Banks have access to the world market where they collect savings foreigners and from the domestic worker households. The exact amount of foreign savings and the economy's net foreign assets (NFA) are explicitly determined by the world interest rate, r. Banks invest in loans to entrepreneurial firms and in the standard production sector.[6] The banking sector is assumed to be perfectly competitive. However, there are financial frictions that restrict the amount of funds allocated to entrepreneurial firms, and the remaining funds are automatically invested in the standard production sector. In equilibrium, the world interest rate, r, which is also the deposit rate, equals the after tax rate of return to capital in the standard production sector:

$$r = (1-\tau_p)r_c.$$

[6] Another interpretation is that the worker households hold shares of the firms in the standard production sector and receive the after tax rate of return to capital.

2.4 Government

The government taxes both labor income and entrepreneurial income, and uses the revenues to finance an exogenously given stream of government consumption G_t. Following Benabou (2000) and Heathcote, Storesletten, and Violante (2017), we assume progressive taxation of labor income. Specifically, the taxes paid on labor income, $\tau(y)$, is determined as follows:

$$\tau(y) = y - \lambda y^{1-\gamma}.$$

where $y = w\varepsilon_j \mu_j z_s$ is before tax labor income of the worker. Here, λ is a constant that controls the level of taxes, and γ controls the progressivity of the tax function.

The tax function for entrepreneurial income, $\tau_k(\pi)$, is assumed to take a similar form:

$$\tau_k(\pi) = \pi - \lambda_k \pi^{1-\gamma_k}$$

where λ_k and γ_k control the level and the progressivity of the tax function.

The tax function parameters for both labor income and entrepreneurial profits are calibrated to mimic the actual U.S. tax rules, which is discussed in detail in Section 3.4. The government runs a pay-as-you-go social security program that pays benefits $SS(z_f)$. Social security benefits are assumed to be a function of the parent's lifetime earning ability, z_f. In addition, the government budget is balanced in each period via the lump-sum transfer, κ.

2.5 Stationary Equilibrium

The state vector in this economy is given by $x = (a, \mu, z_s, z_f, \theta, d))$ where a is asset holdings, μ is the idiosyncratic labor productivity, z_s and z_f represent the lifetime labor ability, θ the entrepreneurial productivity shock, and d whether or not the parent is alive. Let $\{X_j(x)\}_{j=1}^{T}$ represent time-invariant measures of households and $I_j(x)$ be a policy function for the occupational choice where $I_j(x) = 1$ indicates the choice of being an entrepreneur and $I_j(x) = 0$ indicates the choice of being a worker.

The aggregate capital in the standard production sector can be specified as:

$$K = \sum_{j,x} a_j(x)(1 - I_j(x))X_j(x) - \sum_{j,x} l(x)I_j(x)X_j(x) - NFA \tag{11}$$

where $\sum_{j,x} a_j(x)(1 - I_j(x))X_j(x)$ represents the savings of worker households and $\sum_{j,x} l(x)I_j(x)X_j(x)$ are the loans given to entrepreneurial households. Note that NFA is the net foreign assets position of the economy, and thus $-NFA$ represents the savings from the foreigner in the domestic banking sector. In an open economy model, the NFA position is explicitly determined by the world interest rate, r. In the following quantitatively analysis, we are particularly interested in how an exogenous change in the interest rate and thus in NFA (mimicking the U.S. net capital inflow and declining interest rate) affects the economy.

The aggregate labor used in the standard production sector is given by:

$$N = \sum_{j,x} \varepsilon_j \mu_j z_s (1 - I_j(x)) X_j(x) - N_e, \tag{12}$$

where N_e is the amount of labor employed in the entrepreneurial sector:

$$N_e = \sum_{j,x} n_e(x) I_j(x)) X_j(x). \tag{13}$$

A stationary *recursive competitive equilibrium* is defined as follows: Given a fiscal policy (G, τ, τ_k, SS), and a net foreign asset position NFA, a stationary recursive competitive equilibrium is a set of value functions $\{V_j(x), V_j^w(x), V_j^e(x)\}_{j=1}^T$, households' decision rules $\{c_{sj}(x), c_{fj}(x), a_{j+1}(x), I_j(x), l(x)\}_{j=1}^T$, time-invariant measures of households $\{X_j(x)\}_{j=1}^T$ with the state vector $x = (a, \mu, z_s, z_f, \theta, d))$, and relative prices $\{w, r\}$, such that:

1. Given the fiscal policy and prices, households' decision rules solve households' decision problem in equation 6,7, and 8.

2. Factor prices solve the firm's profit maximization policy by satisfying equations 9 and 10.

3. Individual and aggregate behavior are consistent, that is, equations 11, 12, and 13 are satisfied.

4. The measures of households satisfy:

$$X_{j+1}(a', \mu', z_s, z_f, \theta', d') = \sum_{\{x:a'\}} \Omega_\theta(\theta, \theta')\Omega(\mu, \mu')\Lambda_j(d, d')X_j(x), \text{for } j < T,$$

$$X_1(a', \mu', z_s^{'}, z_s, \theta', 1) = \sum_{\{x:a'\}} \Omega_\theta(\theta, \theta')\overline{\Omega}(\mu')\Omega_z(z_s, z_s^{'})X_T(x)$$

   where $a' = a_{j+1}(x)$ is the optimal assets in the next period and $\Lambda_j(1,1)$ represents the survival probability of an age j individual to age $j+1$.[7] $X_1(a', \mu, z_s^{'}, z_s, \theta', 1)$ represents the measure of new families in the dynasty where $\overline{\Omega}(\mu')$ is the distribution from which the labor productivity is drawn for a new adult child.

5. The government's budget holds. That is, total taxes collected are equal to the sum of government

[7] $\Lambda_j(0,1) = 0; \Lambda_j(1,0) = 1 - \Lambda_j(1,1); \Lambda_j(0,0) = 1.$

expenditures, G, social security transfers, and transfers, and is given by:

$$G + \sum_{j,x} dSS(z_f)X_j(x) + \sum_{j,x} \kappa(x)X_j(x) = \sum_{j,x} \{\tau(w\varepsilon_j\mu_j z_s)[1 - I_j(x)] + \tau_k[\pi(a,\theta)]I_j(x)\} X_j(x)$$
$$+\tau_p(Y - wN - \delta K). \tag{14}$$

3 Calibration

We calibrate the benchmark model so that the steady state of the economy matches some key moments of the U.S. economy in the 2000s. Our calibration strategy consists of two stages. In the first stage, we predetermine the values of some standard parameters based on the existing literature or independent estimations. In the second stage, we calibrate the rest of the parameters to simultaneously match some key empirical moments constructed from the data.

3.1 Demographics, Preferences, and Labor Income

A model period is assumed to be one year. A newborn in this economy is 30 years old and can live up to 100 years old. The value of T is set to 35. That is, at age 65 an individual and his/her economically active 30-year-old child forms a new household. Retirement is mandatory at age 65 after which individuals face mortality risk. The transition matrix $\Lambda_j(d, d')$ is calibrated using the U.S. life table.[8]

The utility function is assumed to take the CRRA form. That is,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where σ is set to 2.0 and the subjective time discount factor, β is set to 0.96, both standard values used in the literature.

All individuals face the same labor income process. The deterministic age-specific labor efficiencies are taken from Hansen (1993). Idiosyncratic labor productivity is modeled as a combination of a persistent and a transitory shock following Heathcote et al. (2010). That is, the logarithm of idiosyncratic labor productivity, $log(\mu_j)$, is determined by:

$$log(\mu_j) = \eta_j + \nu_j,$$

$$\eta_j = \rho\eta_{j-1} + \omega_j.$$

[8]Table 7 summarizes the mortality risk at five-year age intervals over the lifecycle calculated from the U.S. life table by the authors.

Here, η_j is the persistent component with ω_j representing the persistent shock, ν_j is the transitory shock, and ρ the persistence of the income shock.[9] Both persistent and transitory shocks follow a normal distribution, and their variances are denoted by σ_ω^2 and σ_υ^2.

In the benchmark calibration, we set the parameter values for the labor productivity shock process based on Heathcote et al. (2010). Specifically, we set the value of ρ to 0.9733 and set the variances of the two shocks based on the estimates for the year 2000. That is, $\sigma_\omega^2 = 0.0212$ and $\sigma_\upsilon^2 = 0.0872$.[10] We discretize the labor productivity shock to a 10-state Markov Chain with 5 states for the persistent shock, and 2 states for the transitory shock.[11] The resulting values of μ and its corresponding 10x10 transition matrix $\Omega(\mu, \mu')$ are reported in Table 8. Labor efficiencies for high and low-ability individuals are calibrated to match those of college and non-college graduate males as in Fuster, İmrohoroğlu, and İmrohoroğlu (2003) where the values for the transition probabilities are chosen so that the proportion of full-time male workers that were college graduates was 28% and the correlation between the permanent component of income of parents and children is 0.4. This calibration strategy implies that $\Omega_z(H, H) = 0.57$ and $\Omega_z(L, L) = 0.83$.

3.2 Technology and Entrepreneurship

There are two production sectors the standard production sector and the entrepreneurial sector. The standard production sector features competitive firms that produce goods using a Cobb-Douglas production function:

$$Y = AK^\alpha N^{1-\alpha}.$$

In our benchmark economy, we assume that the share of capital in the Cobb-Douglas production function, α, is equal to 0.33, and the capital depreciation rate, δ, is 5%. Both are commonly used values in the quantitative macro literature. The depreciation rate and the share of capital in the entrepreneurial sector are assumed to be the same as those in the standard production sector. TFP (total factor productivity) parameters A and A_e are also assumed to be the same, and their values are chosen so that output per household is normalized to one.

The entrepreneurial sector features a decreasing returns to scale production technology, where the degree of decreasing return is given by ϕ. As discussed in the model section, the entrepreneurial productivity shock can take three values, 0, θ_l and θ_h, and it follows a 3x3 Markov chain. Therefore, there are in total 9 parameters remaining to be determined for the entrepreneurial sector. That is, the degree of decreasing returns ϕ, the two non-zero entrepreneurial productivity levels θ_l and θ_h, and the 6 parameters governing the corresponding transition matrix. We calibrate them to match the following moments with regard to the population share of entrepreneurs, and the wealth distribution: (1) the shares of total wealth held by

[9] The persistent component at age 1, η_1, is assumed to be equal to ω_1.

[10] Table 12 reports the estimates of the variances for all years from Heathcote et al. (2010).

[11] Following Kopecky and Suen (2010), we discretize the persistent shock using the Rouwhorst (1995) method . The transitory shock is discretized using the standard Tauchen method (1986).

the top 10 percent and the top 5 percent, (2) the shares of total wealth held by five wealth quintiles, (3) the wealth Gini of the U.S., and (4) the population share of entrepreneurs. The resulting values for the decreasing return to scale parameter is 0.65, and the entrepreneurial productivity shocks are 1.3 and 3.4. The corresponding transition matrix is reported in Table 10.

3.3 Banking Sector

We assume that the banking sector is competitive so that the deposit rate is equal to the lending rate and the after tax rate of return implied in the standard production sector. The benchmark model is assumed to be an open economy and the interest rate, r, is exogenously determined by the world interest rate, which is set to be 3% in the benchmark model representing the 2000s. As the entrepreneurial firm can only pledge to repay a share, η, of its own capital in the next period, the bank is willing to lend them up to a limit that their incentive-compatibility constraint holds. We set the value of η to 0.33 based on Cagetti and De Nardi (2006).

3.4 Government Policies

We model the progressive labor income taxation in the fashion of Benabou (2000) and Heathcote et al. (2017). That is, the tax paid on labor income y is determined by the following formula, $\tau(y) = y - \lambda y^{1-\gamma}$. Here the curvature parameter in the tax function, γ, controls the progressivity, and the constant in the tax function, λ, controls the level. We calibrated their values to match the estimated marginal tax rates on labor income in the U.S. (provided in Bhandari and McGrattan (2019)). The resulting value of the progressivity parameter γ is 0.05, which is slightly on the lower side compared to some existing estimates.[12]

We adopt a similar strategy for the progressive taxation on entrepreneurial incomes. That is, we assume the tax function for entrepreneurial incomes also takes the form in the fashion of Benabou (2000), and calibrate the parameter values to match the marginal tax rates on entrepreneurial incomes estimated in Bhandari and McGrattan (2019). The resulting marginal tax rates and their data counterparts are reported in Table 11. As for the tax rate on corporate profit, τ_p, we set it to 23% based on Gravelle (2004).

The social security benefits are a non-linear function of individuals' average past earnings, and the benefit formula is set to mimic the actual rules of the U.S. social security program (see Table 9).[13] Government expenditure, G, is determined endogenously in the equilibrium according to equation 14, and the resulting government expenditure is approximately 14% of total output in our benchmark economy.

[12] For instance, Kaymak and Poschke (2016) estimated this parameter, and obtained a value of 0.08. In the appendix, we conduct counterfactual experiments to evaluate the impact of labor income tax progressivity on wealth inequality.

[13] The same benefit formula are widely use in the literature including Fuster, Imrohoroglu, and Imrohoroglu (2007); Zhao (2014); and others. Note that the social security tax rate is part of the progressive income tax in the model.

3.5 Calibrating to the1980s

There are three changes we make to the benchmark calibration in order to represent the economy in the 1980s and to investigate the role they may have played in the increase in wealth inequality since then. These are the world interest rate, corporate income tax rate, and income risk. According to the IMF data, the real interest rate facing the U.S was around 6-8% in the 1980s.[14] This rate came down gradually over the next several decades, and the average real interest rate in the 2000s was approximately 3%.[15] In computing the steady state designed to represent the 1980s economy, we set the world interest rate to 6%.

Some existing studies have suggested that changes in taxation can be a potential driving force behind the increasing wealth inequality in the U.S.[16] A major change in taxation since the 1980s is the decline in corporate income tax.[17] As Gravelle (2004) computed, the marginal corporate income tax rates was roughly in the range of 30%-40% in the 1980s, and it declined to 23% in 2003. Thus, in the 1980s steady state, we set the tax rate on corporate profit, τ_p, to 35%.[18]

Another often mentioned potential cause of the increasing wealth inequality is the increase in income inequality. An important dimension of the increased income inequality over the last several decades is the increase in labor income risks in the U.S. For instance, Heathcote et al. (2010) documented in the PSID data that the variances of the transitory and persistent labor income risks increased substantially since the 1960s. For the 1980s calibration, we use the estimates for the year 1980 in Heathcote et al. (2010) for the exogenous labor income process. Specifically, we set the variances of the two shocks as follows: $\sigma_\omega^2 = 0.0171$ and $\sigma_\upsilon^2 = 0.062$ and discretize the labor productivity shock to a 10-state Markov Chain in the same fashion as in the benchmark calibration.[19]

Table 1 summarizes the main results of our calibration exercise. Table 2 reports the main targets used in the calibration and the corresponding model counterparts.

[14]The real interest rate is defined as the lending interest rate adjusted for inflation as measured by the GDP deflator. The source of the data on real interest rate is: International Monetary Fund, International Financial Statistics, and data files using World Bank data on the GDP deflator.

[15]The declining interest rate in the U.S. has often been attributed to the "global saving glut" hypothesis proposed by Bernanke (2005). That is, the excess supply of saving from emerging economies and oil exporters (such as China) flowed into the U.S. market and pushed down the market interest rate (see Caballero, Farhi, and Gourinchas (2008), for example). In our benchmark analysis, we treat the decline in interest rate as an exogenous process, and focus on its distributional implications.

[16]See, for example, Kaymak and Poschke (2016) and Hubmer et al. (2018).

[17]During this period of time, the level and progressivity of labor income taxation has been relatively stable (see Kaymak and Poschke, 2016).

[18]It is worth noting that other studies such as Piketty and Saez (2007) also found that corporate taxes have decreased substantially over the past several decades, but they exogenously assume that the tax incidence of corporate taxes is entirely on stock shareholders. In contrast to these studies, in this paper, we specifically model each type of taxes and let the model endogenously determines the tax incidence of corporate tax.

[19]The resulting values of μ and its corresponding 10x10 transition matrix $\Omega(\mu, \mu')$ in the 1980s steady state are available upon request from the authors.

Table 1: Benchmark Calibration

Description	Parameter	Value
Capital income share	α	0.33
Capital depreciation rate	δ	0.05
Risk aversion parameter	σ	2.0
Time discount factor	β	0.96
TFP factor	A,A_e	0.48
Debt limit (share of capital)	η	0.33
Tax function parameter (labor)	λ,γ	0.63,0.05
Tax function parameter (entrepreneur)	λ_k,γ_k	0.8, 0.04
Bank interest rate	r	0.03
Variances of labor productivity shocks	σ^2_ω, σ^2_v	0.0212, 0.0872
Corporate tax rate	τ_p	0.23
Entrepreneurial productivity shock:	θ_l, θ_h	$1.3, 3.4$
Transition matrix for θ	$\Omega_\theta(\theta, \theta')$	see Table 10
Decreasing return to scale parameter	ϕ	0.65
Bank interest rate in the 1980s	r	0.06
Variances of labor productivity shocks in the 1980s	σ^2_ω, σ^2_v	0.0171, 0.062
Corporate tax rate in the 1980s	τ_p	0.35

4 Results

We start this section by examining the properties of the steady state reflecting the economic conditions in the U.S. in the 2000s. Next, we introduce changes in the tax system, interest rates, and income inequality to mimic the economic conditions in the 1980s and examine how these three factors contribute to understanding the wealth distribution in the 1980s. Lastly, we compute the transition from the steady state in the 1980s towards the future and compute the impact of the changes in these factors in accounting for the rise in wealth inequality since the 1980s.

4.1 Steady State Properties of the Benchmark Economy

Table 2 summarizes some of the main characteristics of the steady state economy and their data counterparts. This economy generates a wealth Gini of 0.79, an average household saving rate of 6.4%, and a capital-output ratio of 2.8. Top wealth shares generated by the model are consistent with the U.S. data. As documented in the 2007 SCF data by Kuhn and Rios-Rull (2016), the top 10% of Americans hold 71.4% of total wealth. This number is 60.3% for the top 5% and 33.6% for the top 1% of the wealth distribution in the data. In the benchmark model, the share of total wealth held by the top 1% is 36.8%, and the shares for the top 5% and the top 10% are 63.3% and 72.6%, respectively. The benchmark model also matches the rest of the wealth distribution fairly well. In the model, the shares of total wealth held by the first and second quintiles are only 0.5% and 2.2%, while the same number is 82.7% for the fifth quintile. In the SCF data, the shares of total wealth held by the first two quintiles are -0.2% and 1.1%, and 83.4% for the fifth quintile.

Table 2: Key Statistics: Benchmark Model vs. Data

Statistic	Data	Steady State (2000s)
Bank interest rate (r)	3%	3%
Corporate income tax rate (τ_p)	23%	23%
HH saving rate	7-8%	6.4%
NFA	-43%	-55%
Capital-output ratio	2.8	2.8
Popu share of entrepreneurs	7%	7.2%
Wealth Gini	0.82	0.79
Wealth share by each quintile		
1st	-0.2%	0.5%
2nd	1.1%	2.2%
3rd	4.5%	4.4%
4th	11.2%	10.2%
5th	83.4%	82.7%
Wealth share of the top 10%	71.4%	72.6%
Wealth share of the top 5%	60.3%	63.3%
Wealth share of the top 1%	33.6%	36.8%

Data source: Kuhn and Rios-Rull (2016)

The benchmark model is also capable of generating a reasonable income distribution without directly calibrating to it (see Table 3). Here, income includes labor income, interest income, and entrepreneurial income. In the benchmark model, the shares of total income held by the first to fifth quintiles are 4.9%, 9.3%, 13.5%, 20.5%, and 51.5%, respectively. These numbers are close to what has been observed in the U.S. data except the top quintile.[20] As documented in the SCF data, the shares of total income held by each quintile are 2.8% , 6.7%, 11.3%, 18.3%, and 60.9%.

Table 3: Income Distribution: Model v.s. Data:

Income Distribution: Share of Total (in %)								
	Quintiles					Top Shares		
	1st	2nd	3rd	4th	5th	10	5	1
Data	2.8	6.7	11.3	18.3	60.9	47.1	36.9	21.0
Benchmark	4.9	9.3	13.5	20.5	51.5	37.3	27.5	15.9

Data source: Kuhn and Rios-Rull (2016)

It is worth noting that while our benchmark model matches the income and wealth distributions in the U.S. well, it has other properties that have been difficult to capture in standard lifecycle models such

[20]Note that our benchmark calibration does not target the income distribution. In sensitivity analysis, we try an alternative calibration strategy that better matches the top shares of the income distribution by directly targeting at them.

as a positive relationship between lifetime income levels and saving rates. The positive cross-sectional relationship, that is the rich saving a larger share of their income, has been well documented in the literature (see for example, Dysnan, Skinner, and Zeldes (2004), and Straub (2018)). Table 4 reports the saving rates by each current income quintile as well as the saving rates by permanent income (lifetime income) quintile in the benchmark model.[21] As shown in the top panel of the table, the saving rate increases substantially across the current income distribution. While the average household saving rate is −48.5% for the bottom current income quintile, it is approximately 25.0% for the top quintile. The bottom panel of Table 4 presents the saving rates by permanent income generated in our model. Similarly, the benchmark model is capable of generating a positive relationship between the saving rate and permanent income. In the benchmark model, the average household saving rate is −5.9% for the bottom permanent income quintile, while it is approximately 15.1% for the top quintile.

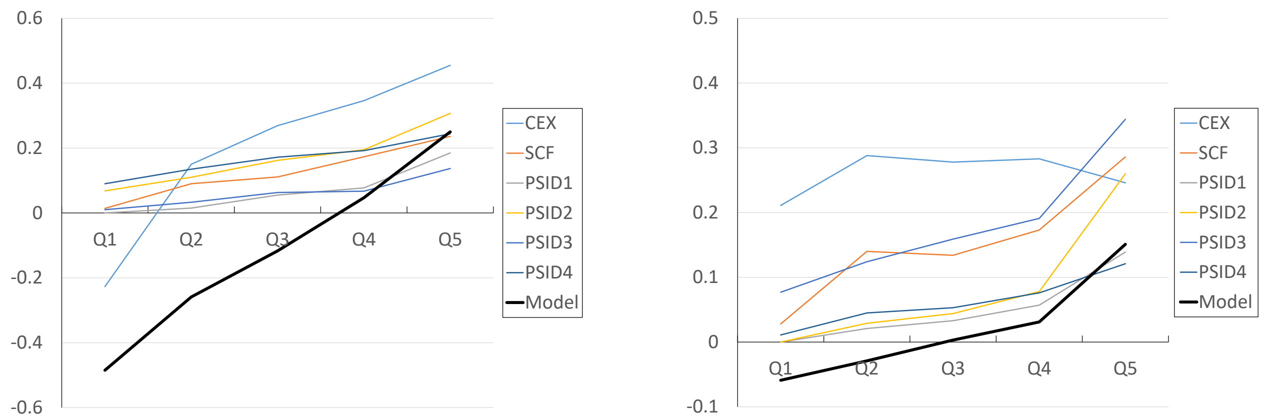
Table 4: Household Saving Rates by Current and Permanent Income:

	Current Income Quintiles					
	1	2	3	4	5	All
Benchmark	−48.5%	−25.9%	−11.7%	4.7%	25.0%	6.4%
	Permanent Income Quintiles					
	1	2	3	4	5	All
Benchmark	−5.9%	−2.9%	0.3%	3.1%	15.1%	6.4%

Figure 1(a) and Figure 1(b) display the saving rates by current income and by permanent income quintiles from various data sources from Dynan et al. (2004) as well as the findings from our benchmark model. While the level of the saving rates vary significantly across different data sets, all display an increase across income quintiles that is also present in the model economy.

[21] These results are calculated from simulating a large number of households from the steady state of the benchmark model. Permanent income is defined as total income over the entire working period.

Figure 1: Saving Rate by Income Quintiles: Model vs. Data



(a) Current Income

(b) Permanent Income

Note: Data source is from Dynan et al. (2004).

In our model, there are multiple factors that lead to differences in saving rates across different income levels. First, returns to wealth are heterogeneous since entrepreneurs can receive a higher return on capital than what is available as the deposit rate through the banks to the workers. Second, intergenerational links via altruistic preferences lead to differences in saving rates across household types where the income of the parent and the children differ from each other.

We do find, through counterfactual experiments, that intergenerational links play an important role in shaping the positive saving rate-income relationship in the model. Intergenerational links also play an important role in the model generated wealth inequality. In an experiment where individuals do not derive utility from the well-being of their children, wealth Gini declines from 0.79 to 0.72.[22]

4.2 Wealth Inequality Across Steady States

There has been in a significant increase in wealth inequality in the United States since the 1980s. According to the data constructed by Saez and Zucman (2016), the share of wealth held by the top one percent in the U.S. has increased from 24.3% in 1980 to 39.5% in 2010. A similar trend is observed in the SCF data where the top one percent wealth share increased from 29.9% in 1989 to 33.6% in 2007.

[22]We present results of a set of counterfactual experiments in the Appendix.

In this section, we change the calibration of the benchmark economy along three dimensions to mimic the 1980s and compute what the steady state would have looked like in the 1980s. Specifically, we set the interest rate equal to 6%, the corporate income tax rate to 35%, and calibrate the exogenous labor income process using the income risk estimates for 1980 from Heathcote et al. (2010). We compare the steady state differences in the wealth distribution between the 1980s and the 2000s and examine the contribution of the three factors to these differences. Table 5 presents the results from the steady state analysis. Note that the model economy is calibrated to match the wealth distribution in the 2000s, so it is not surprising that the share of total wealth held by the top 10%, 5%, and 1% in the model matches the data fairy well for the 2000s. The model economy's results for the 1980s, however, are obtained by introducing differences from the benchmark calibration only in three components. Nevertheless, the share of wealth held by the top 10% and 5% in the 1980s is remarkably close to their counterparts in the data. The share of the top 1% is somewhat lower than its data counterpart, 20.4% in the model versus 29.9%-24.3% in the data. The three factors combined generate a significantly lower wealth inequality in the 1980s compared to the 2000s. The model-generated wealth share for the top 1%, for example, is 20.4% in the 1980s compared to 36.8% in the 2000s.

The last three rows in Table 5 present counterfactual experiments designed to understand the role of the changes in the interest rate, corporate tax rate, and income risk in generating the lower wealth inequality obtained for the 1980s. In the first counterfactual experiment, labeled "1980 Interest Rate," the only change that is applied to the benchmark calibration mimicking the 2000s is the higher interest rate of 6%. Wealth inequality is significantly lower in this case. For instance, the wealth share held by the top 1% in this counterfactual economy is only 18.7% compared to 36.8% in the 2000s, and the wealth shares held by the top 10% is 64%, compared to 72.6% in the 2000s. The substantially different wealth distributions in this counterfactual relative to the benchmark economy suggest that the decline in the interest rate has a significant impact on the increase in wealth inequality from the 1980s to the 2000s. In this model, interest rates affect different types of households very differently. Entrepreneurial households benefit from lower financing costs and increase their investments while worker households face lower returns to their savings as the interest rate declines. These opposing effects lead to higher inequality in wealth in the 2000s.

Table 5: Wealth Inequality: Steady State Analysis

	Share of Total Wealth Held By		
	Top 10%	Top 5%	Top 1%
Data: SCF - Saez & Zucman			
2007; 2010	71.4 – 75.7	60.3 – 62.6	33.6 – 39.5
1989; 1980	67.1 – 67.1	54.2 – 50.7	29.9 – 24.3
Model: Steady State			
2000s	72.6	63.3	36.8
1980s	67.1	56.0	20.4
Counterfactuals			
I. 1980 Interest rate	64.0	52.1	18.7
II. 1980 Corporate tax	73.8	64.7	37.4
III. 1980 Income risk	76.0	68.5	40.5
Model: Transition Path			
2010: Along the Transition	74.8	64.6	27.2
1980 (Initial Steady State)	67.1	56.0	20.4

Note: Data is from two sources. SCF data is for 2007 and 1989, Saez and Zucman (2016) data is for 2010 and 1980.

In the second counterfactual experiment, we set the tax rate on corporate profits to 35%, approximately the average level in the 1980s, which is higher than the tax rate used in the benchmark economy, 23%, representing the 2000s.[23] We find that wealth inequality is slightly higher in the economy with a higher corporate income tax. This finding is different from the conventional wisdom that often says that lower corporate tax rates tend to reduce wealth inequality. The reason for this finding is that corporate income tax not only decreases the interest rate but also reduces the wage rate via general equilibrium effects. In this counterfactual experiment, the equilibrium wage rate is 0.41, approximately 4% lower than that in the benchmark economy. It is worth noting that the implication of corporate income taxes in an open economy like our benchmark model can be different from that in a closed economy. While the endogenously-generated tax incidence of corporate income tax is largely on workers in an open economy, it would be shared by capital owners and workers in a closed economy. As a sensitivity analysis, we also repeat this counterfactual experiment in a closed economy setting and find that while the results are numerically different, the impact of a higher corporate income tax on wealth inequality is still quantitatively small. Thus, according to these results, the decrease in the corporate tax rate between the 1980s and the 2000s is not responsible for the increase in wealth inequality.

In the third counterfactual, "1980 Income risk," the only change that is made to the benchmark economy is the use of the income risk estimates for the year 1980 from Heathcote et al. (2010). Interestingly,

[23]It is worth noting that other studies such as Piketty and Saez (2007) also found that corporate taxes have decreased substantially over the past several decades, but they exogenously assume that the tax incidence of corporate taxes is entirely on stock shareholders. In contrast to these studies, in this paper, we specifically model each type of taxes and let the model endogenously determines the tax incidence of corporate tax.

lower income risks results in higher wealth inequality. The share of total wealth held by the top 1%, for example, increases to 40.5% compared to the steady state results in the 2000s of 36.8%. This is because of the relationship between labor income risk and precautionary savings. High labor income risk increases precautionary saving by workers, and this effect tends to be more profound among poorer individuals. Thus, the increase in income risk between the 1980s and 2000s is also not responsible for the increase in wealth inequality in this framework.

The conclusion that emerges from these counterfactual experiments is the important role played by the decline in the interest rate in accounting for the increase in wealth inequality between the 1980s and 2000s.

The quantitative strategy so far has been to compare and analyze the steady-state implications of the potential drivers for the rising wealth inequality in the U.S. While the steady-state comparison strategy is transparent and computationally less demanding, the steady state analysis does not fully capture the transitory impact of these factors during the transition dynamics. In the rest of the section, we explore the transitional implications of these drivers.

4.3 Wealth Inequality Along the Transition Path

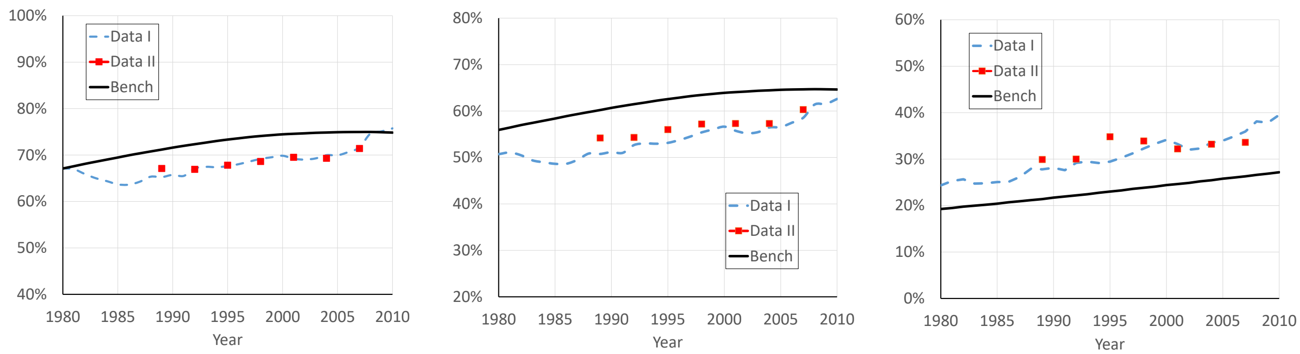
This section presents the time path of the wealth distribution starting from the initial steady state and along the transition path to the new steady state. The initial steady state in 1980 is shocked by imposing changes in three factors: the decline in interest rate, corporate income tax, and the increase in income risk. The transition is assumed to take 150 years. The actual data from 1980 to 2010 on interest rates, corporate income tax rates, and income risk, provided in Table 12, are used under the assumption of perfect foresight for all these factors.[24] The wealth distribution, especially the top wealth shares, along the transition path generated by the model are compared to the U.S. data to evaluate if the model is capable of accounting for the rise in wealth inequality between 1980 and 2010.

Next, the driving forces behind the rise in wealth inequality are evaluated by running counterfactual experiments to isolate the effect of the decline in interest rate and corporate income tax and the increase in income risk on the wealth distribution between 1980 and 2010.

[24] Chen, İmrohoroğlu, and İmrohoroğlu (2006) and İmrohoroğlu and Zhao (2018, 2020) show that the perfect foresight assumption does not have a large impact on the quantitative implications of this type of a model.

Figure 2: Top Wealth Shares along the Transition Paths: Model vs. Data

(a) Share of Wealth Held by Top 10% (b) Share of Wealth Held by Top 5% (c) Share of Wealth Held by Top 1%



Note: Data I represents the data from Saez and Zucman (2016), and Data II is SCF data.

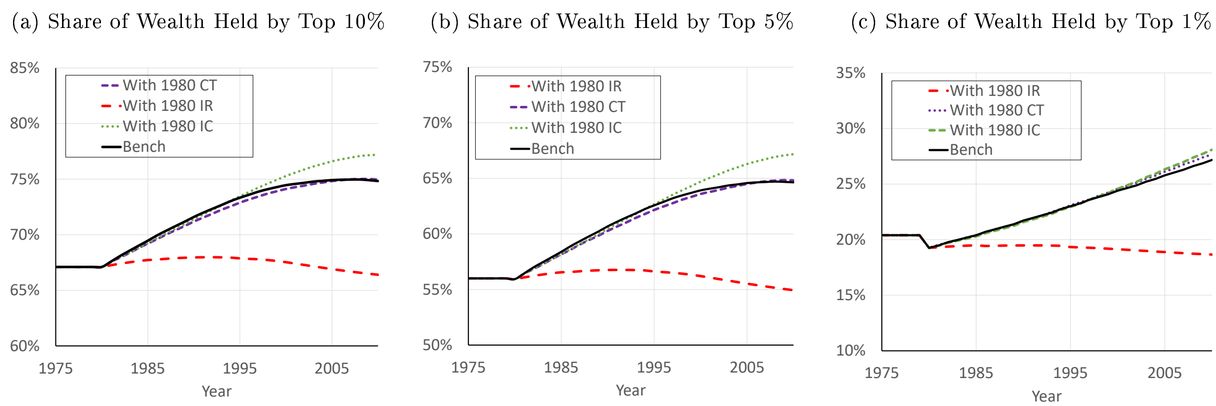
Figure 2 displays the top wealth shares generated by the benchmark economy versus the data starting in 1980. The wealth shares of the top 10% and 5% increase rather quickly in the model over time. In the data, the share of total wealth held by the top 10% is 75.7% in 2010. In the model, it reaches 74.8% in 2010. The top 5% holds 62.6% of total wealth in the data and 64.6% in the model by 2010. Overall, the model captures 89.5% to 72.5% of the increase in the wealth shares of these two groups between 1980 and 2010 in the data.[25] The share of wealth held by the top 1% in the model does not increase as fast as in the data. By 2010, the share of wealth held by the top 1% reaches 27.2% in the model while in the data it is 39.5%. This group's wealth share reaches 36% by 2060.

Figure 3 provides the results from a set of counterfactual experiments designed to investigate the forces behind the increase in the wealth shares along the transition. In each counterfactual experiment, one exogenous variable is set to its level in the 1980s. For example, in the experiment labeled "With 1980 IR," the interest rate is kept constant at its 1980 level of 6% all through the transition path. Figure 3 displays the results for this case where the rise in top wealth shares completely disappears for all three wealth shares. In the experiment labeled "With 1980 CT," the corporate income tax rate is set to its 1980 value of 35% all along the transition path. In this case, the top wealth shares do not differ from the benchmark in any visible way, suggesting that the decrease in corporate income tax does not play a significant role in shaping the rise in wealth inequality. Lastly, in the experiment labeled "With 1980 IC," the increase in the income risk that took place during this time is assumed away by keeping the income risk along the transition the

[25]The share of wealth held by the top 10% and 5% increases by 8.6 and 11.9 percentage points between the steady states representing the 1980s and 2000s (from 67.1% to 75.7% for the top 10% and from 50.7% to 62.6% for the top 5%). Along the transition path of the model, the same shares increase by 7.7 and 8.7 percentage points (67.1% to 74.8% for the top 10% and 55.9% to 64.6% for the top 5%), capturing 89.5% to 72.5% of the increase in the data (the last two rows of Table 5 shows the transition results).

same as it was in the 1980s. Similar to the findings in the steady state results, this factor also does not make a significant impact on the transition path found for the benchmark economy. These counterfactual experiments together reveal again that an important cause of the increase in wealth inequality captured in this framework is the decline in interest rate.[26] The intuition for this finding is simple. The decline in interest rate hurts worker households by reducing the rate of return on their saving while it benefits entrepreneurial households by lowering their financing cost.

Figure 3: Top Wealth Shares Along the Transition Paths: Counterfactuals



Note: the case labeled "Bench" represents the benchmark transition path with changes in all three factors: income risk, corporate income tax, and interest rate. The case labeled "With 1980 IR" represents the counterfactual transition path without the decline in interest rate, the case labeled "With 1980 CT" is the counterfactual transition path without changes in corporate income tax, and the case labeled "With 1980 IC" represents the counterfactual transition path without changes in income risk.

4.4 Welfare Implication

In this section, we examine the welfare implications of the decline in interest rate by comparing two transition path economies: the benchmark transition path featuring the decline in interest rate, and the counterfactual transition path with no change in interest rate (i.e., constant 6% interest rate over time). Specifically, we compare the expected lifetime utility of a newly formed household (i.e., at age 1) in 1980, at the beginning of the transition path in these two economies. As shown in Table 6, a newborn household in 1980 has a higher expected lifetime utility on average in the benchmark transition path economy ($EV_1 = -89.1$) than in the counterfactual without the decline in interest rate ($EV_1 = -92.0$). In terms of consumption equivalent

[26]If all three factors are kept constant at their 1980 levels, the top wealth shares would remain constant along the transition path. We do not plot this case on Figure 3 to save space.

variation (CEV), the welfare gain from the decline in interest rate is on average 3.2% of consumption.[27] In addition, we find that the welfare gain from the decline in interest rate is much larger for worker households. While households without entrepreneurial skills experience a welfare gain equivalent to 3.3% of consumption, those with entrepreneurial skills only enjoy a small welfare gain, that is equivalent to 1.6% of consumption.

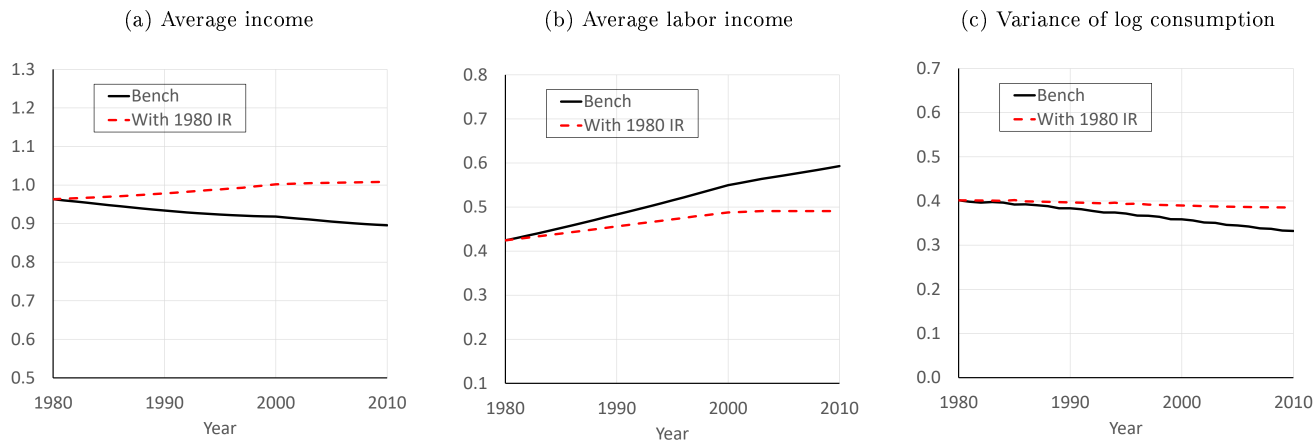
Table 6: Welfare Consequences: Newborns in 1980

	Benchmark	Counterfactual No decline in interest rate
Lifetime utility (value Function)		
w/o entrepreneurial skills	-91.0	-94.0
w/ entrepreneurial skills	-67.6	-68.7
All	-89.1	-92.0
Welfare gain (CEV)		
w/o entrepreneurial skills	3.3%	n.a.
w/ entrepreneurial skills	1.6%	n.a.
All	3.2%	n.a.
Wage Rate:		
1980	0.339	0.357
2010	0.427	0.357
Interest Rate:		
1980	6.0%	6.0%
2010	3.0%	6.0%

The reason for the positive welfare consequence is partly due to the general equilibrium effects. That is, the decline in interest rate leads to a rise in the equilibrium wage rate, which in turn increases labor income. As labor income tends to account for an larger share of income for poor workers among the lower tail of the wealth distribution, the increase in labor income improves consumption smoothing and reduces consumption variation. Figure 4 displays average income (labor and interest income), average labor income, and variance of log consumption over time in the two transition path economies. As can be seen, the variance of log consumption declines substantially while average labor income increases in the transition path with the decline in interest rate. This point can also be seen in Figure 5, which plots the evolution of the consumption (and income) shares of the top and bottom wealth quintiles along the transition paths. In the benchmark transition path, the consumption (and income) share of the bottom quintile increases while that of the top quintile decreases. In other words, while wealth inequality increases, the consumption (and income) distribution by wealth partition compresses. This finding highlights the intuition behind the welfare gain from the decline in interest rate, and it is largely because of the increased labor income, which is more concentrated among the lower tail of the wealth distribution.

[27] That is, to gain the same expected lifetime utility as in the benchmark economy, a newborn household in the counterfactual economy without the decline in interest rate has to increase consumption in each period by 3.2%.

Figure 4: Understanding the Welfare Consequences of the Decline in Interest Rate



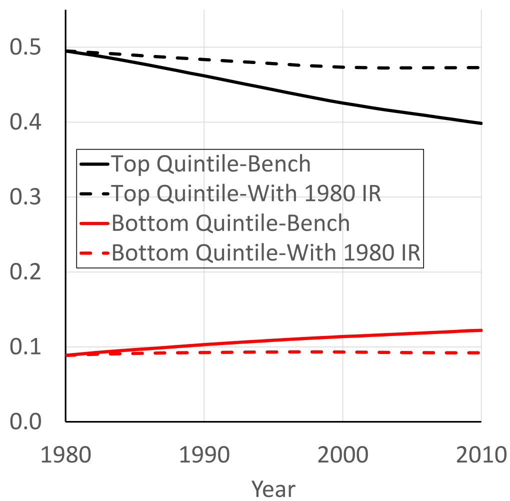
Note: the case labeled "Bench" represents the benchmark transition path with changes in all three factors: income risk, corporate income tax, and interest rate. The case labeled "No Change in IR" represents the counterfactual transition path without the decline in interest rate.

5 Conclusion

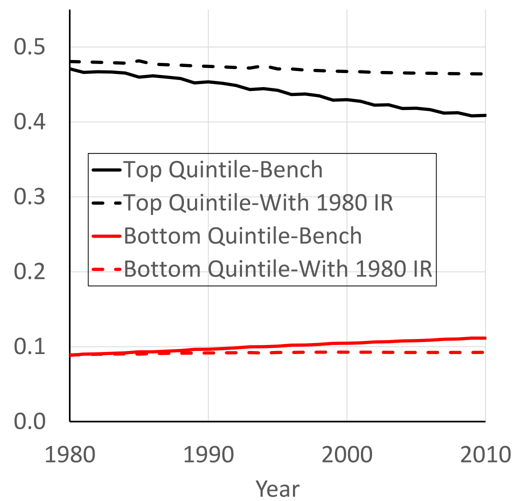
In this paper, we show that a dynamic general equilibrium model that properly captures intergenerational links and occupation choice (particularly working vs. entrepreneurship) is capable of generating a highly unequal wealth distribution as in the U.S. data. We use the model to quantitatively evaluate the potential role played by several factors in accounting for the rise in wealth inequality in the U.S. since the 1980s. We find that the decline in the interest rate plays a major role in accounting for this increase as it affects the two types of households very differently. Entrepreneurial households benefit from lower financing costs and increase their investments while worker households face lower returns to their savings as the interest rate declines. Other changes such as the decline in corporate profit tax plays a less significant role. Interestingly, we find that the increase in labor income risks (as documented in Heathcote et al. 2010), an important dimension of the increasing income inequality, reduces wealth inequality. This is because rising labor income risk increases precautionary saving by workers who tend to be wealth-poor. It is worth noting that while the decline in the interest rate substantially increases wealth inequality, it is welfare-improving, and the welfare gain is larger for worker households than entrepreneurs. This is largely due to the general equilibrium effects, the decline in interest rate raising the equilibrium wage rate, which benefits workers more than the entrepreneurs and thus improves consumption smoothing.

Figure 5: Understanding the Welfare Consequences of the Decline in Interest Rate

(a) Income shares based on wealth partition



(b) Consumption shares based on wealth partition



Note: the case labeled "Bench" represents the benchmark transition path with changes in all three factors: income risk, corporate income tax, and interest rate. The case labeled "With 1980 IR" represents the counterfactual transition path without the decline in interest rate.

References

[1] Bernanke, B.S., 2005. "The global saving glut and the U.S. current account deficit," Speech 77, Board of Governors of the Federal Reserve System (U.S.).

[2] Benabou, R., 2000. “Unequal Societies: Income Distribution and Social Contract,” The American Economic Review, 90 (1), 96–129.

[3] Benhabib, J., Bisin, A., and M. Luo. 2019. “Wealth Distribution and Social Mobility in the US: A Quantitative Approach.” American Economic Review, 109(5), pages 1623-47.

[4] Benhabib, J., Bisin, A., and M. Luo. 2017. “Earnings Inequality and Other Determinants of Wealth Inequality,” American Economic Review: Papers & Proceedings, 107 (5), pages 593–597.

[5] Benhabib, J., Bisin, A., and S. Zhu. 2011. “The Distribution of Wealth and Fiscal Policy in Economies With Finitely Lived Agents,” Econometrica, 79 (1), pages 123–157.

[6] Bhandari, A and E. R. McGrattan, 2019. “Sweat Equity in U.S. Private Business,” Federal Reserve Bank of Minneapolis, Staff Report 560.

[7] Caballero R.J., Farhi, E. and P.-O. Gourinchas. 2008. “An Equilibrium Model of “Global Imbalances” and Low Interest Rates.” American Economic Review, 98(1), pages 358-393.

[8] Cagetti, M. and M. De Nardi. 2006. “Entrepreneurship, Frictions, and Wealth,” Journal of Political Economy, vol. 114, n. 5, pages 835-870.

[9] Castaneda, A., Diaz-Gimenez, J., and J.-V. Rios-Rull. 2003. “Accounting for the U.S. earnings and wealth inequality,” Journal of Political Economy, 111(4), Pages 818–857.

[10] Chen, K., İmrohoroğlu, A., and S. İmrohoroğlu, 2006. “The Japanese Saving Rate”, American Economic Review, 96(5): 1850-1858.

[11] Cooke, A., Lee, H., and K. Zhao. 2017. “Houses divided: a model of intergenerational transfers, differential fertility and wealth inequality,” Working paper.

[12] De Nardi, M. 2004. “Wealth inequality and intergenerational links,” Review of Economic Studies, 71(3), Page 743–768.

[13] De Nardi, M., and F. Yang. 2016. “Wealth inequality, family background, and estate taxation,” Journal of Monetary Economics, Volume 77, Pages 130-145.

[14] Dynan, K.E., Skinner, J. and S. P. Zeldes. 2004. “Do the Rich Save More?”Journal of Political Economy, vol. 112, no. 2.

[15] Fuster, L., A. İmrohoroğlu, and S. İmrohoroğlu, 2003. "A welfare analysis of social security in a dynastic framework", International Economic Review, vol. 44(4), pages 1247-1274, November.

[16] Fuster, L., A. İmrohoroğlu, and S. İmrohoroğlu, 2007. "Elimination of Social Security in a Dynastic Framework", Review of Economic Studies, vol. 74(1), pages 113-145.

[17] Gabaix, X., J.-M. Lasry, P.-L. Lions, and B. Moll. 2016. "The Dynamics of Inequality", Econometrica, 84(6): 2071-2111.

[18] Gravelle, J. G., 2004. "The Corporate Tax: Where Has It Been and Where Is It Going?" National Tax Journal, Vol. LVII, No. 4.

[19] Greaney, B., 2020. "The Distributional Effects of Uneven Regional Growth", Working Paper.

[20] Hansen, G D, 1993. "The Cyclical and Secular Behaviour of the Labour Input: Comparing Efficiency Units and Hours Worked," Journal of Applied Econometrics, vol. 8(1), pages 71-80.

[21] Heathcote, J., Storesletten, K., and G. L. Violante, 2010. "The Macroeconomic Implications of Rising Wage Inequality in the United States," Journal of Political Economy, 118 (4), 681–722.

[22] Heathcote, J., Storesletten, K., and G. L. Violante, 2017. "Optimal Tax Progressivity: An Analytical Framework," Quarterly Journal of Economics, Volume 132, Issue 4, Pages 1693–1754.

[23] Hubmer, J., P. Krusell, and A. A. Smith, 2018. "A Comprehensive Quantitative Theory of the US Wealth Distribution," working paper, Yale University.

[24] İmrohoroğlu, A. and K. Zhao. 2018. "The Chinese Saving Rate: Long-Term Care Risks, Family Insurance, and Demographics." Journal of Monetary Economics, Volume 96, Pages 33-52.

[25] İmrohoroğlu, A. and K. Zhao. 2020. "Household Saving, Financial Constraints, and the Current Account in China." International Economic Review, Volume 61, Issue 1, Pages 71-103.

[26] Kaymak, B., & Poschke, M.,2016. "The Evolution of Wealth Inequality over Half a Century: The Role of Taxes, Transfers and Technology," Journal of Monetary Economics, Volume 77, Pages 1–25.

[27] Kim, T., 2020. "The World's Banker: On the Rise in U.S. Wealth Inequality," Working Paper.

[28] Kopecky, K., and R.M.H. Suen, 2010. "Finite State Markov-chain Approximations to Highly Persistent Processes", Review of Economic Dynamics, Volume 13, Issue 3, Pages 701-714.

[29] Kuhn, M., and J.-V. Ríos-Rull, 2016. "2013 Update on the U.S. Earnings, Income, and Wealth Distributional Facts: A View from Macroeconomic Modelers", Federal Reserve Bank of Minneapolis Quarterly Review, Vol. 37 No. 1.

[30] Laitner, J. 1992. "Random Earnings Differences, Lifetime Liquidity Constraints, and Altruistic Intergenerational Transfers." Journal of Economic Theory, 58, 135–170.

[31] Mendoza, E.G., V. Quadrini, and J.-V. Rios-Rull. 2009. "Financial Integration, Financial Development and Global Imbalances." Journal of Political Economy, 117(3).

[32] Mohaghegh, M. 2019. "Evolution of Wealth Inequality In the U.S., Entrepreneurship and Financial Intermediation." Working paper.

[33] Piketty, T., Saez, E.,2007. "How progressive is the U.S. federal tax system? A historical and international perspective." Journal of Economic Perspectives, 21(1), 3–24.

[34] Quadrini, V., 2000. "Entrepreneurship, Saving, and Social Mobility," Review of Economic Dynamics, 3 (1), 1–40.

[35] Rouwenhorst, K.G., 1995. "Asset pricing implications of equilibrium business cycle models." In: Cooley, T.F. (Ed.), Frontiers of Business Cycle Research. Princeton University Press, Princeton, NJ, pp. 294–330.

[36] Saez, E. and G. Zucman, 2016. "Wealth Inequality in the United States Since 1913: Evidence from Capitalized Income Tax Data," Quarterly Journal of Economics, 131 (May), 519–578.

[37] Straub, L. 2018. "Consumption, Savings, and the Distribution of Permanent Income", MIT working paper.

[38] Tauchen, G. 1986. "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions." Economics Letters, 20(2), 177-181.

[39] Zhao, K. 2014. "Social security and the rise in health spending," Journal of Monetary Economics, Volume 64, Pages 21-37.

Appendix

5.1 Additional Tables and Figures

Table 7: Survival Probabilities:

Age	<65	65	70	75	80	85+
Surv.	1	.0.9847	0.9765	0.9623	0.9379	0.8934

Data source: the U.S. life table.

Table 8: Idiosyncratic Labor Productivity

$\mu:$	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7	μ_8	μ_9	μ_{10}
	0.156	0.294	0.554	1.045	1.970	0.508	0.957	1.805	3.404	6.419
$\Omega(\mu,\mu')$	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7	μ_8	μ_9	μ_{10}
μ_1	0.474	0.026	0.001	0.000	0.000	0.474	0.026	0.001	0.000	0.000
μ_2	0.006	0.474	0.019	0.000	0.000	0.006	0.474	0.019	0.000	0.000
μ_3	0.000	0.013	0.474	0.013	0.000	0.000	0.013	0.474	0.013	0.000
μ_4	0.000	0.000	0.019	0.474	0.006	0.000	0.000	0.019	0.474	0.006
μ_5	0.000	0.000	0.001	0.026	0.474	0.000	0.000	0.001	0.026	0.474
μ_6	0.474	0.026	0.001	0.000	0.000	0.474	0.026	0.001	0.000	0.000
μ_7	0.006	0.474	0.019	0.000	0.000	0.006	0.474	0.019	0.000	0.000
μ_8	0.000	0.013	0.474	0.013	0.000	0.000	0.013	0.474	0.013	0.000
μ_9	0.000	0.000	0.019	0.474	0.006	0.000	0.000	0.019	0.474	0.006
μ_{10}	0.000	0.000	0.001	0.026	0.474	0.000	0.000	0.001	0.026	0.474

Table 9: Social Security Benefit Function

Average lifetime earnings	Replacement rate
$[0, 0.2\bar{y})$	90%
$[0.2\bar{y}, 1.25\bar{y})$	33%
$[1.25\bar{y}, 2.47\bar{y})$	15%
$[2.47\bar{y}, \infty)$	0%

Note: $\bar{y}$ is the average lifetime earnings of the economy.

Table 10: Transition Matrix for Entrepreneurial Productivity θ

$\theta :$	θ_1	θ_2	θ_3
	0	$\theta_l = 1.3$	$\theta_h = 3.4$
$\Omega_\theta(\theta, \theta')$	θ_1	θ_2	θ_3
θ_1	0.985	0.015	0
θ_2	0.165	0.8	0.035
θ_3	0	0.03	0.97

Note: $\bar{y}$ is the average lifetime earnings of the economy.

Table 11: Marginal Tax Rates on Entrepreneurial Income: Model v.s. Data:

	Marginal Tax Rates	
Entrepreneurial Income brackets	Data	Benchmark
0-0.153	14%	n.a.
0.153-0.304	18.3%	15.0%
0.304-0.912	20.1%	19.0%
0.912-2.667	23.5%	21.7%
2.667-5.727	26.2%	24.9%
5.727-9.104	26.9%	26.4%
9.104+	28%	28.9%

Data source: Bhandari and McGrattan (2019).

Table 12: Data for Simulations

Year	Interest rate	Corporate Income Tax	Variance of Income Risk	
			persistent shock	transitory shock
1980	0.06	0.35	0.0171	0.062
1981	0.059	0.345	0.0173	0.063
1982	0.058	0.340	0.0175	0.065
1983	0.057	0.334	0.0177	0.066
1984	0.056	0.329	0.0179	0.067
1985	0.055	0.324	0.0181	0.068
1986	0.054	0.319	0.0183	0.070
1987	0.053	0.313	0.0185	0.071
1988	0.052	0.308	0.0187	0.072
1989	0.051	0.303	0.0189	0.073
1990	0.050	0.298	0.0192	0.075
1991	0.049	0.293	0.0194	0.076
1992	0.048	0.287	0.0196	0.077
1993	0.047	0.282	0.0198	0.078
1994	0.046	0.277	0.0200	0.080
1995	0.045	0.272	0.0202	0.081
1996	0.044	0.267	0.0204	0.082
1997	0.043	0.261	0.0206	0.083
1998	0.042	0.256	0.0208	0.085
1999	0.041	0.251	0.0210	0.086
2000	0.04	0.246	0.0212	0.087
2001	0.039	0.240	0.0212	0.087
2002	0.038	0.235	0.0212	0.087
2003	0.037	0.23	0.0212	0.087
2004	0.036	0.23	0.0212	0.087
2005	0.035	0.23	0.0212	0.087
2006	0.034	0.23	0.0212	0.087
2007	0.033	0.23	0.0212	0.087
2008	0.032	0.23	0.0212	0.087
2009	0.031	0.23	0.0212	0.087
2010	0.03	0.23	0.0212	0.087
..	..	..	..	..
New SS	0.03	0.23	0.0212	0.087

5.2 Determinants of Steady State Wealth Inequality

It is well known that the unequal wealth distribution in the U.S., particularly the extremely high top wealth shares, are hard to be reconciled in standard heterogeneous-agent models. Why is our benchmark steady state model capable of generating such an unequal wealth distribution? In our benchmark model, there are multiple factors that increase the inequality in wealth. First, returns to wealth are heterogeneous since entrepreneurs can receive a higher return to capital than what is available as the deposit rate through the banks. Second, intergenerational links via altruistic preferences lead to differences in saving rates across household types and contribute to wealth inequality in the steady state.

In this section, we examine the factors that potentially contribute to the steady state wealth inequality in the benchmark model. To examine the contribution of each factor, we conduct counterfactual experiments in which we shut down each factor while keeping everything else the same as in the benchmark. The results are reported in Table 13.

We find that an important factor contributing to the top wealth shares is entrepreneurship. In the first experiment, when we assume away the opportunity of becoming entrepreneurs, the wealth inequality in the model substantially decreases with the top 1% only holding 11% of total wealth, and the top 5% and the top 10% holding 34% and 48% of total wealth, respectively. The wealth Gini drops to 0.63. The impact of entrepreneurship on wealth inequality is partly from the uncertainty in entrepreneurial productivity, which implies that entrepreneurial investments have differential returns. As shown in the second experiment, if we only assume away the uncertainty in entrepreneurial productivity, wealth inequality decreases by significantly less, with the wealth Gini dropping to 0.73. It is interesting to note that earnings risks do not contribute to wealth inequality in this model. In the third experiment, when assuming away earnings risk, the top wealth shares substantially increase; for instance, the top 1% now holds 59%. This is because earnings risk induces precautionary saving and this impact is more pronounced among individuals concentrated in the lower part of the wealth distribution.[28] Earnings risks come from both persistent shock and transitory shock, and in the fourth and fifth experiments, we find that both shocks contribute to the impact of earnings risks. Other factors such as corporate tax and progressive income tax play a minor role in shaping the wealth distribution.

The last experiment examines the role of intergenerational links in shaping the long-run wealth inequality. To shut down intergenerational links, in this experiment, we assume that individuals do not derive utility from the well-being of their children., and they exit the economy at age T so that individuals never overlap with their adult children. As shown in the last line in Table 13, when intergenerational links are shut down, the wealth Gini declines significantly, from 0.79 to 0.72. The share of total wealth held by the top 10% decreases from 72.6% to 62.1%, the wealth share held by the top 5% decreases from 63.3% to 51.4%, and the top 1% wealth share remains similar to the benchmark level.

Overall, our counterfactual experiments suggest that the key factors contributing to the steady-state wealth inequality are entrepreneurship and intergenerational links.

[28]The negative impact of earning risks on wealth inequality is also found in Hubmer et al. (2018).

Table 13: Determinants of Steady-State Wealth Inequality in the Benchmark Model

	Share of Total Wealth (in %)			
	Top 10%	Top 5%	Top 1%	Wealth Gini
Benchmark	72.6%	63.3%	36.8%	0.79
Counterfactual Experiments:				
1.Entrepreneurship	48.4%	33.9%	11.0%	0.63
2.Entrepreneurial risk	62.7%	48.0%	31.9%	0.74
3.Earnings risk	95.8%	92.2%	59.2%	0.96
4.Transition earning shock	83.6%	74.6%	44.5%	0.89
5.Persistent earning shock	81.4%	75.8%	46.1%	0.84
6.Corporate income tax	71.0%	62.5%	36.0%	0.78
7.Progressive tax	71.8%	62.9%	36.2%	0.79
8. Intergenerational links	62.1%	51.4%	37.1%	0.72

5.3 Saving Rates by Income

Why do the rich save more? In our model, there are multiple factors that lead to differences in saving rates across different lifetime income levels. First, returns to wealth are heterogeneous since entrepreneurs can receive a higher return to capital than what is available as the deposit rate through the banks. Second, intergenerational links via altruistic preferences lead to differences in saving rates across household types where the income of the parent and the children differ from each other.

To evaluate the quantitative importance of these factors, here we conduct two additional counterfactual experiments. Specifically, we consider: (1) a model with no entrepreneurship, and (2) a model with neither entrepreneurship nor intergenerational links. Figure 6 plots the relationships between the saving rate and permanent income for each of the two counterfactual experiments. As can be seen in Case I, while entrepreneurship is responsible for the high saving rates of the top quintiles, it does not account for the positive saving rate-income relationship. When only entrepreneurship is shut down, the positive relationship between the saving rate and income remain similar to that in the benchmark. However, as shown in Case II, when intergenerational links are also assumed away, the significant positive relationship between the saving rate and permanent income mostly disappears. The results from these two counterfactual cases suggest that intergenerational links play an important role in shaping the positive saving rate-income relationship.

5.4 Sensitivity Analysis

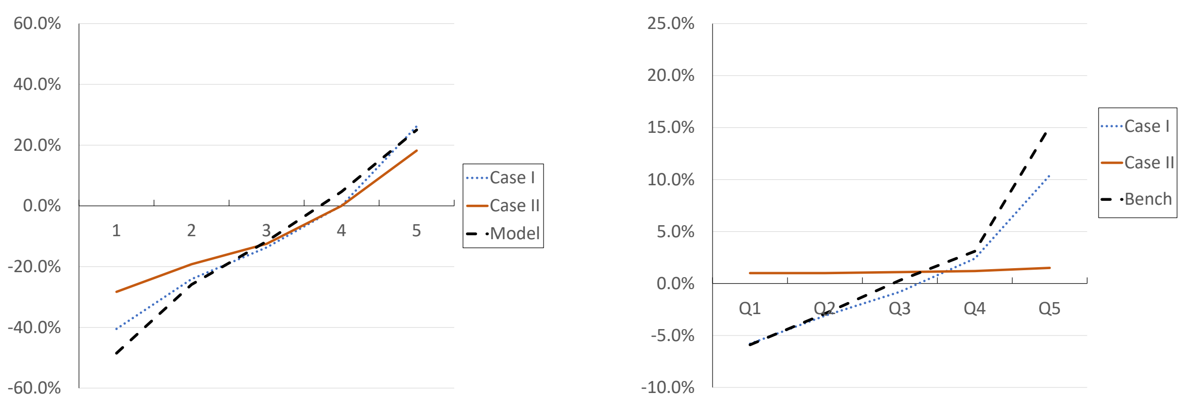
As briefly discussed in Section 4, the implication of corporate income taxes in an open economy like our benchmark model can be different from that in a closed economy. While the endogenously generated tax incidence of corporate income tax is largely on workers in an open economy, it would be shared by capital owners and workers in a closed economy.

In this section, we conduct sensitivity analysis with regard to the impact of a higher corporate income tax

Figure 6: Saving Rate by Income Quintiles: Counterfactual Experiments

(a) Current Income

(b) Permanent Income



Note: Case I has no entrepreneurship, and Case II has neither entrepreneurship nor intergenerational links.

in a closed economy setting. Specifically, we repeat the counterfactual experiment with a corporate income tax rate of 35% while keeping the economy closed.[29] As expected, the incidence of corporate income tax is partly shared by capital owners. The after-tax return capital (or the interest rate r) declines slightly from 3% to 2.6% as the corporate income tax rate is raised from 23% to 35%. The corresponding decrease in wage rate is also smaller, with the equilibrium wage rate in this counterfactual experiment being 0.42, only 1% lower than that in the benchmark economy.

We find that the resulting wealth inequality in this experiment is only slightly higher than that in the experiment with the open economy with a wealth Gini of 0.805. This is largely due to the slightly lower after-tax interest rate in this closed economy setting. Thus, our sensitivity analysis confirms that the change in corporate income tax is not responsible for the rise in wealth inequality since the 1980s.

[29]To keep the economy closed, we simply exogenously keep the NFA position of the economy fixed at the level as in the benchmark economy when checking the market-clearing conditions.