The impact of the correlation between health expenditure and survival probability on the demand for insurance

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Abstract

This paper studies the effects of health shocks on the demand for health insurance and annuities, along with precautionary saving in a dynamic life-cycle model. I argue that when the health shock can simultaneously increase health expenses and reduce longevity, rational agents would neither fully insure their uncertain health expenses nor fully annuitize their wealth because the correlation between health expenses and longevity provides a self-insurance channel for both uncertainties. That is, when the agent is hit by a health shock (which simultaneously increases health expenses and reduces longevity), she can use the resources originally saved for consumption in the reduced period of life to pay for the increased health expenses. Since the two uncertainties partially offset each other, the precautionary saving generated in the model should be smaller than in a standard model without the correlation between health expenses and longevity. In a quantitative life-cycle model calibrated using the Medical Expenditure Panel Survey dataset, I find that the health expenses are highly correlated with the survival probabilities, and this correlation significantly reduces the demand for actuarially fair health insurance, while its impact on the demand for annuities and precautionary saving is relatively small.

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1. Introduction

It is well known that health expenses in developed countries have risen dramatically over the last several decades and are projected to continue rising in the near future. For instance, the US aggregate health expenses rose from 5.2% of GDP in 1960 to 16% of GDP in 2007, and are projected to be 25% in 2025 and 37% in 2050. Importantly, health expenses are extremely volatile and a significant portion of these expenses are not insured (out-of-pocket expenses). Meanwhile, a large cohort of baby boomers will be retiring in the next decade. Hence, it is important to understand how health shocks affect retirees’ lifetime financial planning.

In this paper, I develop a dynamic life-cycle model with health shocks and use it to study the effects of health shocks on the demand for health insurance and annuities, and precautionary saving. In particular, I focus on health shocks that simultaneously increase health expenses and reduce longevity, and how they affect retirees’ lifetime financial planning decisions.

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1 The 1960 and 2007 numbers are from OECD Health Data 2009. The projected numbers are from the Congressional Budget Office.
Many types of health shocks have simultaneous effects on health expenses and longevity. For instance, entering into long-term care not only significantly increases health expenses, but also reduces survival probabilities to the future (Sinclair and Smetters, 2004; Kopecky and Koreshkova, 2009, etc.) Hurd et al. (2001) found that a variety of health conditions (e.g., cancer, heart disease) can reduce survival probabilities. Similar results are found for the general concept of a health shock, a health status change. De Nardi et al. (2010) documented in the AHEAD data that conditional on permanent income, gender, and age, people in good health status spend around 50% less on health care annually than those in bad health status, but they expect to live about 3 years longer than those in bad health.

I argue that when health shocks can simultaneously increase health expenses and reduce longevity, several interesting results can be obtained. First, utility-maximizing agents would neither fully insure their uncertain health expenses nor fully annuitize their wealth, even when these insurance policies are actuarially fair and there is no bequest motive. Second, when the insurance markets for uncertain health expenses and uncertain longevity are missing, the precautionary saving generated by these uncertainties may be smaller than in a model without the correlation between health expenses and longevity.

The intuition behind these results is simple. The simultaneous effect of health shocks on health expenses and longevity provides agents with a self-insurance channel for both uncertain health expenses and uncertain longevity. When the agent is hit by a health shock (which simultaneously increases health expenses and reduces longevity), she can use the resources originally saved for consumption in the reduced period of life to pay for the increased health expenses. As a result, agents would neither fully insure their health expenses nor fully annuitize their wealth. Similarly, when the insurance markets for uncertain health expenses and uncertain longevity are missing, since the two uncertainties partially offset each other through the correlation between them, the precautionary saving generated in the model would be smaller than in a model without the correlation between health expenses and longevity.

To assess the quantitative importance of the above described results, I develop a quantitative dynamic life cycle model with uncertain health expenses and uncertain longevity, and calibrate the correlation between the two uncertainties using the Medical Expenditure Panel Survey (MEPS) dataset. I find that current health expenses are highly correlated with the conditional survival probabilities to the next period. Then, I run computational experiments in the calibrated model to quantify the impact of the correlation between health expenses and survival probabilities. I find that its impact on the demand for health insurance is quantitatively large, while its impact on the demand for annuities and precautionary saving is relatively small.

Some existing studies have also implicitly captured the correlation between health expenses and longevity. For instance, De Nardi et al. (2010) captured the correlation between health expenses and survival probabilities via including health status as a state variable in their model. Kopecky and Koreshkova (2009) partially captured the correlation via modeling a nursing home shock. However, these studies usually do not model the decisions to buy health insurance and annuities, and thus have not explored the implications of this correlation for the demand for health insurance and annuities, which is the main goal of this paper. In addition, I assume that the survival probability is directly conditioned on the current health expense, and measure the magnitude of the correlation between health expenses and survival probabilities from the MEPS dataset.

This paper also contributes to the literature that aims to understand why households do not buy more private health insurance and annuitize their wealth. I find that the correlation between health expenses and survival probabilities is an important reason why many individuals do not buy more private health insurance, but it cannot explain the non-annuitization puzzle.

This paper is related to Sinclair and Smetters (2004) who have also studied the implications from the simultaneous effect of health shocks on health expenses and longevity. In a quantitative OLG model, they show that the simultaneous effect of health shocks on health expenses and longevity reduces the demand for annuities via numerical simulations. In this paper, I show that the correlation between health expenses and survival probabilities also reduces the demand for health insurance, and this effect is quantitatively more important than the effect on the demand for annuities. In addition, I provide new implications for precautionary saving, and show that the impact from the correlation between health expenses and longevity may be different across the income distribution.

This paper is also related to a recent growing literature that uses quantitative dynamic models to study the impact of uncertain health expenses on precautionary saving. In this paper, I argue that the correlation between health expenses and survival probabilities provides agents a self-insurance channel, and thus may be important for understanding the impact of uncertain health expenses on precautionary saving.

The rest of the paper is organized as follows. In Section 2, I present a simple example to illustrate the intuition. In Section 3, I study an analytical model and derive some theoretical results. I develop the full quantitative dynamic life cycle model in Section 4.1 and present the main quantitative results in Section 5. I conclude in Section 6.

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3 Hubbard et al. (1995), De Nardi et al. (2010), Kopecky and Koreshkova (2009), etc.
2. A simple model

In this section, I present a simple model to illustrate the intuition behind the main findings of this paper. Here I only look at the problem after retirement. Assume that an agent with endowment $W$ faces the following two-period expected utility maximization problem:

$$
\max_{C_1(h), C_2(h)} E[U(C_1(h)) + S(h)U(C_2(h))]
$$

subject to

$$
W - M(h) - C_1(h) = C_2(h), \quad \forall h,
$$

$$
C_1(h) \geq 0 \quad \text{and} \quad C_2(h) \geq 0, \quad \forall h
$$

Here $U(C)$ represents the utility flow derived from consumption $C$, $M$ is the health expense, and $S$ is the survival probability to period 2. The agent receives a health shock, $h$, at the beginning of period 1. When it is a bad shock, i.e., $h = h_b$, the agent needs to pay health expenses $M(h_b) = W/2$, and she will not survive to period 2 for sure, i.e., $S(h_b) = 0$. When it is a good shock, the agent needs to pay no health expense, i.e., $M(h_g) = 0$, and she will survive to the second period for sure, i.e., $S(h_g) = 1$. For simplicity, the discount factor and the gross interest rate are both equal to one. To have a meaningful problem, I also assume that the expected health expenses are less than the initial endowment, i.e., $E[M(h)] < W$. Agents face the following expected utility maximizing problem:

$$
\max_{t \in [0, T(h)]} E[\int_0^{T(h)} U(C(t, h)) \, dt],
$$

subject to

$$
W - M(h) = \int_0^{T(h)} C(t, h), \quad \forall h.
$$

$$
C(t, h) \geq 0, \quad \forall h, t.
$$

Here $U(\cdot)$ satisfies the following conditions: $U' > 0$, $U'' < 0$, $U'' > 0$, and the Inada conditions. $C(t, h)$ represents the consumption at time $t$, conditional on the health shock, $h$, which has the following properties: $h = h_g$ (good shock) with a probability of $1 - P$, and $h = h_b$ (bad shock) with a probability of $P$. The lifetime health expenses and the longevity are determined by the health shock in the following way, $M(h_g) = 0$, $M(h_b) = M$, and $T(h_g) = T$, $T(h_b) = \delta T$, where $0 < \delta < 1$.

Since both the discount factor and the gross interest rate are equal to one, it is obvious that rational agents will choose a flat consumption path after the health shock. That is, $C(t, h) = C(t', h)$, for any $t, t' \in [0, T(h)]$. Using $C(h)$ to represent the constant consumption per period, the above utility maximizing problem is simplified to the following problem:

$$
\max_{C(h)} E[T(h) U(C(h))]
$$

subject to

$$
W - M(h) = T(h)C(h), \quad \forall h
$$

$$
C(h) \geq 0, \quad \forall h.
$$

Assuming that neither annuities nor health insurance are available, the optimal solution for the above problem can be easily obtained. That is, $C(h_g) = W/T$ and $C(h_b) = (W - M)/\delta T$. As can be seen, health insurance or annuities before the health shock is revealed can be welfare-improving as long as the following condition holds:

$$
C^*(h_g) = \frac{W}{T} \neq \frac{W - M}{\delta T} = C^*(h_b)
$$
thus,
\[ M \neq W(1 - \delta) \] (8)

3.1. Health insurance

Now I consider agents’ demand for health insurance in this model. Assume that the annuity market is closed, but agents have access to actuarially fair health insurance. That is, the price of health insurance with a coinsurance rate of \( \delta \) is \( q_\delta = PIM \). Agents maximize their expected lifetime utility by choosing the optimal coinsurance rate, \( I^* \). That is, they face the following expected utility-maximizing problem:
\[
\max_{C(h)} \mathbb{E}[T(h)U(C(h))] \tag{9}
\]
subject to
\[
W - M(h) - PIM + M(h)I = T(h)C(h), \quad \forall h,
\]
\[
C(h) \geq 0, \quad \forall h, \quad \text{and} \quad I \geq 0.
\] (10)

Let us study this problem in two different scenarios.

(1) \( M \leq W(1 - \delta) \). As shown in Eqs. (7) and (8), even without any health insurance, agents already have a higher consumption per period after a bad health shock than after a good shock. Therefore, in this scenario, agents do not need any health insurance, i.e., \( I^* = 0 \) (corner solution). The intuition behind this result is simple. If the health expenses (i.e., \( M \)) are not larger than the resources freed up from a reduction in longevity (i.e., \( W(1 - \delta) \)), health insurance is not needed, as the self-insurance channel itself is enough to insure against the risk.

(2) \( M > W(1 - \delta) \). In this scenario, there exists an interior solution for \( I \). After substituting the budget constraint into the objective function, the following first order condition (FOC) can be obtained:
\[
-(1 - P)T U \left( \frac{W - PIM}{T} \right) P M \delta T U \left( \frac{W - M - PIM + IM}{\delta T} \right) \left( M - PM \right) \frac{1}{\delta T} = 0 \] (11)

Rearranging the above equation and solving for \( I \):
\[
I^* = \frac{1 - W}{1 - (1 - \delta)} \] (12)

The above equation describes the optimal solution for \( I^* \). From this equation, the following propositions can be obtained:

**Proposition 1.** (1) The optimal health coinsurance rate, \( I^* \), is less than 1. In other words, agents do not choose to fully insure their health expense risk. (2) The optimal health coinsurance rate, \( I^* \), decreases as the reduction in life expectancy increases, i.e., \( \partial I^*/\partial \delta < 0 \), \( I^* \) increases as the probability of getting a bad shock increases, i.e., \( \partial I^*/\partial P > 0 \), and \( I^* \) decreases as the endowment increases, \( \partial I^*/\partial W < 0 \).

**Proof.** As for statement (1), since the expected health expense is less than the initial endowment, the following inequation holds, \( \mathbb{E}[M|h] = PM < W \). Rearranging and multiplying both sides of this inequation by \((1 - \delta)\), I obtain \( P(1 - \delta) < W/M(1 - \delta) \). As a result, \( 1 - (W/M)(1 - \delta) < 1 - P(1 - \delta) \), and thus \( I^* = (1 - (W/M)(1 - \delta))/(1 - P(1 - \delta)) < 1 \). Statement (2) can be simply obtained by taking the first order derivative of Eq. (12) with respect to \( \delta \), \( P \), and \( W \), respectively. □

3.2. Annuities

Now I consider agents’ demand for annuities in this model. Assume that the health insurance market is closed, but agents have access to actuarially fair annuities. That is, the price of annuity policy that pays \( A \) per period when she happens to live longer than expected. Therefore, agents do not need any annuity, i.e., \( A^* = 0 \).
The intuition behind this result is simple. If the health expense saved is larger than the resources needed for the extra years of life, no annuity is needed.

(2) \( M < W(1 - \delta) \). In this case, agents need annuities to insure against the risk of outliving their resources (interior solution). After substituting the budget constraints into the objective function, the following FOC can be obtained:

\[
(1 - P)TU \left( \frac{W}{T} - (P\delta A + (1 - P)A) + A \right) \left[ 1 - (P\delta + (1 - P)) \right] + P\delta TU \left( \frac{W - M}{\delta T} - \left( P\delta A + (1 - P)A \right) + A \right) \left[ 1 - \left( P + \frac{(1 - P)}{\delta} \right) \right] = 0
\]

Rearranging the above equation and solving for \( A \):

\[
A^* = \frac{W - M}{P\delta T + (1 - P)T} \tag{15}
\]

The above equation describes the optimal annuity level, and the price of this annuity policy is

\[
q^*_A = (P\delta T + (1 - P)T)A^* = W - \frac{M}{(1 - \delta)} \tag{16}
\]

As can be seen, the annuitized wealth (measured by \( q^*_A \)) is less than the total wealth available after the health shock (\( W \) or \( W - M \)). In other words, agents do not fully annuitize their wealth. I summarize the main properties of the optimal annuitization decision in the following proposition.

**Proposition 2.** (1) Agents do not fully annuitize their wealth, i.e., \( q^*_A < W - M \) or \( W \). (2) The annuitized wealth (measured by \( q^*_A \)) increases as the initial wealth increases, i.e., \( \partial q^*_A / \partial W > 0 \). (3) The annuitized wealth decreases as the health expenses increase, i.e., \( \partial q^*_A / \partial q < 0 \). (4) The annuitized wealth increases as the reduction in life expectancy increases, i.e., \( \partial q^*_A / \partial (1 - \delta) > 0 \).

**Proof.** Statement (1) is from the assumption \( 0 < \delta < 1 \). Statements (2)-(4) can be easily obtained by taking the first order derivative of Eq. (16) with respect to \( W, M \), and \( 1 - \delta \), respectively. \( \square \)

It may be also interesting to look at another measure of annuitization, the fraction of wealth that is annuitized, which can be measured by \( q^*_A / W = 1 - (M/(1 - \delta))W \). By taking the first order derivative of \( q^*_A / W \) with respect to \( W, M \), and \( 1 - \delta \), respectively, it is easy to see that statements (2)-(4) in Proposition 2 would still hold if \( q^*_A / W \) is used as the measure of annuitization instead of \( q^*_A \).

4 Note that the same result is obtained if \( q^*_A / (W - M) \) is used instead of \( q^*_A / W \).

5 I do not present the derivation here as it is trivial.

6 It is worth noting that annuities and health insurance are in fact insuring against the same risk in the model, but in the opposite direction. Therefore, the optimum can also be achieved by holding both. That is, agents can always increase their holdings of both annuities and health insurance simultaneously and still achieve perfect consumption smoothing because the extra annuities and health insurance offset each other. I rule out this possibility here. In reality, this result is very unlikely to occur because there are entry costs and administrative costs in both markets.
4.1 Calibration

I calibrate the benchmark model to the current US economy. The labor productivity parameters are determined as follows. The logarithm of the individual-specific permanent productivity shock, $\ln c$, is assumed to follow the normal distribution: $N \sim (0, \sigma_c^2)$. I discretize the distribution into five states using the method introduced in Tauchen (1986). Transforming the values back from the logarithms, I get a finite set of consumption floors $\{c_1, c_2, c_3, c_4, c_5\}$, with the corresponding probabilities. The variance of the log of the permanent productivity shock, $\sigma_c^2$, is set to 0.65 such that it is consistent with the empirical estimates of the variance of log annual earnings of men in Heathcote et al. (2010). The age-specific productivities, $\{\epsilon_j\}_{j=1}^R$, are calculated from the earnings data in the Current Population Surveys. The calibrated results on labor productivity are reported in Tables 1 and 2.

Specifically, I use the MEPS dataset to calibrate the health expense shock $m$ and the corresponding survival probability $S(m)$. Specifically, I calibrate the four states for $m$ by breaking down the health expenditure distribution into four bins of sizes

### Table 1

<table>
<thead>
<tr>
<th>Statistic</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$c_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent productivity</td>
<td>0.23</td>
<td>0.48</td>
<td>1.00</td>
<td>2.07</td>
<td>4.27</td>
</tr>
<tr>
<td>Corresponding pop. measure</td>
<td>0.09</td>
<td>0.24</td>
<td>0.35</td>
<td>0.24</td>
<td>0.09</td>
</tr>
</tbody>
</table>

conditional survival probability from age $j$ to $j+1$ is a function of the health expense in age $j$, i.e., $S_j(m_j)$. Ideally, both survival probability and health expense should be modeled as functions of the health condition/status. However, as the information on the dynamics of health conditions is limited in the data, I do not specifically model health conditions here.

Assume that there exists neither health insurance nor annuity in the benchmark model. The utility-maximization problem facing an agent at age $j$ with a permanent shock $\epsilon_j$, health expense $m_j$, and savings $a_j$ can be simply described by the following Bellman equation:

$$V(j, a_j, m_j, \epsilon_j) = \max_{c_j} \left[ \frac{1}{\beta} \frac{c_j^{-\gamma}}{1-\gamma} + \beta S_j(m_j) E[V(j+1, a_{j+1}, m_{j+1}, \epsilon_{j+1})] \right]$$

subject to

$$\left\{ \begin{array}{ll}
    a_{j+1} + c_j + m_j = w_j \gamma_j \epsilon_j (1 - r_s - r_m - r_w) + a_j(1 + r) + b + tr & \text{if } j \leq R \\
    a_{j+1} + c_j + (1 - \kappa_m)m_j = a_j(1 + r) + SS(\epsilon_j) + tr & \text{if } j > R \\
    a \geq 0 & \text{and } c \geq 0
\end{array} \right.$$

Here $V$ is the value function, $r$ is the interest rate, and $b$ is the transfer from accidental bequests, which are assumed to be equally redistributed to the working agents.

There are three government programs: social security, medicare, and the welfare program. Social security imposes a payroll tax $r_s$ on workers and provides annuity payments $SS(\epsilon_j)$ to retirees. The medicare program covers a $\kappa_m$ fraction of health expenses for retirees and imposes a payroll tax $r_m$ on workers. The welfare program guarantees a minimum consumption floor $c$ by providing transfer payments $tr$, which is defined as

$$\left\{ \begin{array}{ll}
    tr = \max [c_j + m_j - w_j \gamma_j \epsilon_j (1 - r_s - r_m - r_w) - a_j(1 + r) - b, 0] & \text{if } j \leq R \\
    tr = \max [c_j + (1 - \kappa_m)m_j - a_j(1 + r) - SS(\epsilon_j), 0] & \text{if } j > R
\end{array} \right.$$

For simplicity, I assume that the prices $\{w, r\}$ are fixed; therefore, a stationary equilibrium can be simply sketched as follows. A stationary equilibrium for a given set of government parameters and prices is a collection of value functions, individual decision rules, a distribution function, and transfer from accidental bequests, such that

1. The value functions and individual decision rules solve the agent’s utility maximization problem.
2. Social security, medicare, and the welfare program are self-financing.
3. The amount of transfers from bequests in each period is equal to the amount of accidental bequests in that period.
4. The population distribution is constant over time.

4.1 Calibration

I calibrate the benchmark model to the current US economy. The labor productivity parameters are determined as follows. The logarithm of the individual-specific permanent productivity shock, $\ln c$, is assumed to follow the normal distribution: $N \sim (0, \sigma_c^2)$. I discretize the distribution into five states using the method introduced in Tauchen (1986). Transforming the values back from the logarithms, I get a finite set of consumption floors $\{c_1, c_2, c_3, c_4, c_5\}$, with the corresponding probabilities. The variance of the log of the permanent productivity shock, $\sigma_c^2$, is set to 0.65 such that it is consistent with the empirical estimates of the variance of log annual earnings of men in Heathcote et al. (2010). The age-specific productivities, $\{\epsilon_j\}_{j=1}^R$, are calculated from the earnings data in the Current Population Surveys. The calibrated results on labor productivity are reported in Tables 1 and 2.

For simplicity, I assume that there is no mortality risk and health expenses before retirement since this paper is mainly concerned about old-age issues. That is, $S_j = 0$ for all $j \leq R$. After retirement, the health expense $m$ is assumed to be governed by a 4-state Markov chain with $m = \{m_1, m_2, m_3, m_4\}$ and the transition matrix $\text{Trans}(x, y) = \text{Prob}(m_{j+1} = m| m_j = m_{j-1})$.

I use the MEPS dataset to calibrate the health expense shock $m$ and the corresponding survival probability $S(m)$. Specifically, I calibrate the four states for $m$ by breaking down the health expenditure distribution into four bins of sizes

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I do so for each five- or ten-year group. Then, I calculate the transition matrices for $m$ for each age group directly from the panel data. The calibrated health expenditure levels are reported in Table 3, and the calibrated transition matrices are reported in the appendix. The corresponding survival probabilities $S_j(m_j)$ can also be calculated from the MEPS dataset, and they are reported in Table 4. As can be seen, the conditional survival probabilities are highly correlated with the current health expenses. For instance, the conditional survival probability to the next year for an agent within the 66–70 age group is 99.1% if her current health expense is below the 50th percentile of the distribution, but her survival probability would decrease to 88.7% if her current health expense is above the 95th percentile of the distribution. Similar results can also be found for other age groups. Note that the main purpose of this paper is to show that this correlation between survival probability and health expenses is important for understanding the agent’s behaviors, i.e., the demand for health insurance and annuities, and precautionary saving.

On the government side, I set the social security tax rate $\tau_s$ to 12.4% according to the SSA data. The benefit formulae $SS(\epsilon_i)$ are assumed to have the same structure as in Fuster et al. (2007) so that the program captures the progressivity of the US Social Security program. I rescale every beneficiary’s benefits so that the social security program is self-financing. According to the CMS data, approximately 50% of the elderly’s health expenditures are paid by medicare, thus I set the medicare coinsurance rate $\kappa_m$ to 0.5. The medicare payroll tax rate $\tau_m$ is endogenously determined by medicare’s self-financing budget constraint. I set the value of the consumption floor $c$ to $2663 based on the estimation in De Nardi et al. (2010), and then endogenously determine the value of $\tau_w$.

As for the prices, the interest rate is set to 4%, and the wage rate $w$ is chosen so that the output per capita in the model is consistent with the US GDP per capita. The value of the discount factor $\beta$ is chosen to match the capital–output ratio in the US, i.e., 3.0. The rest of the parameter values are directly determined based on the estimates in the standard dynamic macroeconomics literature. That is, the elasticity parameter in the CRRA utility function, $\sigma$, is set to 2. Table 5 summarizes the calibrated parameter values.

Since the model is complicated and cannot be solved analytically, I solve it using numerical techniques. Specifically, I solve the decision rules for agents backward from the last period. Some key statistics of the calibrated economy are reported in Table 6. As can be seen, the calibrated model is consistent with the data along most dimensions. For instance, both the output per capita and the capital–output ratio closely match the data. Health expenses per elderly person and the medicare tax rate are on the slightly higher side, which is partly due to that the population structure at steady state does not exactly match the current U.S. population distribution. One exception is the fraction of elderly people on welfare, which is somehow significantly lower than in the data. This is probably due to the simplifying assumptions made in the analysis such as no idiosyncratic earnings shock and exogenous labor supply, which underestimate the risk facing agents in the model. The life cycle profiles of saving and consumption for an average agent in the model are plotted in Fig. 1.

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8 The implied unconditional average survival probabilities at each age are slightly different from those in the US life table from National Vital Statistics Reports. To adjust for this difference, I scale all survival probabilities at each age proportionally.

9 See Attanasio et al. (2008) for a detailed description of medicare.

10 Note that another reason why the medicare tax rate in the model is higher is because I do not model the medicare premiums and therefore a slightly higher tax rate is needed for the medicare program to be self-financing.
5. Main quantitative results

5.1. Health insurance

To understand the impact of the correlation between health expenses and survival probabilities on the demand for health insurance, I now introduce a health insurance market into the benchmark economy. I assume that the health insurance market offers a menu of actuarially fair policies with a coinsurance rate ranging from 0% to 100%. The policy with a coinsurance rate $I$ will cover $I$ fraction of the health expenses uncovered by medicare in each period over the remainder of the holder's life, and meanwhile the holders of the policy are required to pay a premium in each period. For simplicity, I assume that agents have access to the health insurance market only once in their lives, that is, at age $j = R$ (right before retirement). In addition, I assume that agents can only insure uncertain health expenses with those in their own age.
subject to

\[V(j, a_j, m_j, \epsilon_i) = \max_{a_{j+1}, \beta} \left( \frac{C^1 - \sigma}{0 + \beta E(S_j(m_j))V(j+1, a_{j+1}, m_{j+1}, \epsilon_i, I)} \right)\]

The utility-maximization problem facing agents is as follows:

subject to

\[a_{j+1} + c_{j+1} + m_j = w\gamma_j \epsilon_j (1 - \tau_s - \tau_m - \tau_w) + a_j (1 + r) + b + \tau_r\]

\[a \geq 0 \quad \text{and} \quad c \geq 0\]

Note that here agents after retirement would have one more state variable, \(I\), which indicates the type of health insurance policy purchased at age \(j = R\). After retirement, the agent's problem is

subject to

\[a_{j+1} + c_{j+1} + (1 - I)(1 - \kappa_m) m_j + q^I_j = \bar{S}(\epsilon_i) + a_j (1 + r) + \tau_r\]

\[a \geq 0 \quad \text{and} \quad c \geq 0\]

I compute the agent’s problem backward from the last period. The health insurance decisions are reported in Table 7. As can be seen, agents do not choose to buy a health insurance policy with a 100% coinsurance rate, though the health insurance market is frictionless. On average, agents choose to insure 66.3% of their uncertain health expenses. To quantify the impact of the correlation between health expense and survival probabilities on the demand for health insurance, I run a counterfactual computational experiment in which I assume away the correlation between survival probability and health expense. That is, I reset the survival probability for every agent at each age to be the same as the average survival probability at that age, and then I recompute the agents’ health insurance decisions. As also shown in Table 7, now agents on average choose to insure 91.1% of their uncertain health expenses. The comparison between the counterfactual results and the benchmark results suggests that the correlation between health expenses and survival probabilities has a large negative effect on the demand for health insurance. This also suggests that the correlation between health expenses and survival probabilities may be important for understanding the optimal insurance arrangements for health expense shocks.

As argued before, the correlation between health expenses and survival probabilities should have a larger impact on richer agents. Since richer agents tend to consume more per period, for the same reduction in longevity, the resources freed up for them should also be higher, and therefore are more likely to be enough to compensate for the simultaneous increase in health expenses. Table 7 also demonstrates the health insurance decisions by income. As can be seen, the relationship between the coinsurance rate and income in the model is non-monotone. The agents at the two ends of the distribution face the same problem as in the benchmark model. At age \(j = 40\), the agent’s problem is

subject to

\[a_{j+1} + c_{j+1} + m_j = \bar{S}(\epsilon_i) + a_j (1 + r) + \tau_r\]

\[a \geq 0 \quad \text{and} \quad c \geq 0\]

Fig. 1 plots the life cycles of consumption and saving for an average agent in the models with and without health insurance. As can be seen, the life cycle profiles look similar in both models. Agents in the model with health insurance accumulate slightly less savings during the working period and dissave faster after retirement because uncertain health expenses are partially covered by health insurance in this model. For the same reason, the consumption near the end of life is lower in this model because agents keep less precautionary savings until the maximum possible age.

11 Note that all agents at age \(j = 40\) have zero health expense (no pre-existing condition), thus there is no adverse selection in the health insurance market.

12 Note that a complete removal of the welfare program would make the model not well-defined. That is, there exists a tiny fraction of the population which is extremely unlucky (hit by a bad productivity shock and a series of bad health expense shocks) and does not have enough resources to cover its health expenses. As a result, I set the minimum consumption floor in the counterfactual model to $100. As a robustness check, I also explore other values (i.e., $50 and $10) and find that the main results do not significantly change.
Assumption, uncertain health expenses would not affect the demand for annuities due to the liquidity constraint, therefore I retire. In addition, I assume that agents can both buy and sell their annuities in the market. Note that with this the annuity market is designed as follows. The annuities are actuarially fair and they are accessible to any agent after expected longevity. Thus, she can simply use the reduced amount of health expense to cover the consumption in the correlated with the health expense, the agent hit by a lower health expense shock also experiences an increase in her between health expenses and survival probabilities is assumed away, everyone chooses full annuitization. The intuition gets hit by a bad health shock (i.e., a high value of \( q_A \)) in the beginning of Section 5.2, any agent will decide whether to annuitize or not. The results on annuitization can also be explained by using the pricing function for annuities,

\[
q_A(j, m_j) = \frac{\sum_{L=1}^{\infty} \prod_{i=j}^{L-1} S_i(m_i|m_j)}{(1+r)^{L-j}}
\]

In this version of the model, agents face the same problem as in the benchmark model without annuities before retirement (\( j < R \)). After retirement, the utility-maximization problem facing agents can be described as follows:

\[
V(j, q_j, m_j, c_j, A_j) = \max_{q_j, a_{j+1}, A_{j+1}} \frac{c_j^{1-\sigma}}{1-\sigma} + E\left[ S_j(m_j) V(j+1, a_{j+1}, m_{j+1}, c_{j+1}, A_{j+1}) \right]
\]

subject to

\[
a_{j+1} + c_j + (1-\kappa_m) m_j + q_A(j, m_j) A_{j+1} = S_j(c_j) + q_A(j, m_j) A_j + a_j (1+r) + A_j
\]

\[
a \geq 0 \quad \text{and} \quad c \geq 0
\]

Note that there is one more state variable, \( A \), for the agent’s holding of annuities after retirement. In addition, at age \( j=41 \), the value of \( A \) is zero for everyone as the annuity market is not accessible to agents at age \( j < 40 \). Again, I compute the agent’s problem backward from the last period. The results on annuitization are reported in Table 8. Here the measure for annuitization I use is the annuitized wealth as a share of total wealth, that is, \( q_A(j, m_j) A_j \). As shown at the beginning of Section 5.2, \( q_A(j, m_j) \) is decreasing in \( m_j \). That is, the market value of annuity asset would decrease if its holder gets hit by a bad health shock (i.e., a high value of \( m_j \)). This implies that holding annuities would amplify the health expense risks facing agents. In other words, annuities are implicitly negative health insurance. Therefore, agents may not want to fully annuitize their wealth when they face significant health expense risks. Their optimal demand for annuities depends on the tradeoff between the longevity insurance benefit provided by annuities and the loss they generate by amplifying the health expense risks. Note that health expense risks are usually less important for richer agents as they have more savings. Thus, the mechanism emphasized here should have a smaller impact on richer agents. This implication is confirmed by the quantitative results presented in Table 8. As can be seen, the least productive agents annuitize only 64.0% of their retirement wealth, while the most productive agents choose full annuitization.

<table>
<thead>
<tr>
<th>Model economies</th>
<th>( c_1 (%) )</th>
<th>( c_2 (%) )</th>
<th>( c_3 (%) )</th>
<th>( c_4 (%) )</th>
<th>( c_5 (%) )</th>
<th>Average (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>0</td>
<td>87.2</td>
<td>79.5</td>
<td>61.5</td>
<td>30.8</td>
<td>66.3</td>
</tr>
<tr>
<td>No correlation</td>
<td>0</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>91.1</td>
</tr>
<tr>
<td>Benchmark with no welfare program</td>
<td>94.9</td>
<td>87.2</td>
<td>79.5</td>
<td>61.5</td>
<td>30.8</td>
<td>74.8</td>
</tr>
<tr>
<td>No correlation</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

5.2. Annuities

In this section, I introduce an annuity market into the benchmark model, and use the extended model to study the impact of the correlation between health expenses and survival probabilities on the demand for annuities. The structure of the annuity market is designed as follows. The annuities are actuarially fair and they are accessible to any agent after retirement. In addition, I assume that agents can both buy and sell their annuities in the market. Note that with this assumption, uncertain health expenses would not affect the demand for annuities due to the liquidity constraint, therefore I can identify the impact of the correlation between health expenses and longevity on the demand for annuities. Since the market is frictionless, the price of an annuity policy should be conditioned on both age \( q_A \) and current health expense \( m_j \). That is, the price of a policy paying one dollar annuity in each period over the rest of the life is described as

\[
q_A(j, m_j) = \frac{\sum_{L=1}^{\infty} \prod_{i=j}^{L-1} S_i(m_i|m_j)}{(1+r)^{L-j}}
\]

The health insurance market is closed in this section so that I can focus on the demand for annuities.

14 In reality, annuities may be not as liquid as in the model and individuals cannot freely sell back their annuities in the market. Thus, the model here may have overstated the optimal demand for annuities. If agents are not allowed to sell back annuities in the market, their demand for annuitization would be even lower than what has been found in the model.

15 For instance, in the calibrated model, the price of $1 annuity at age \( j=41 \) is $11.4 for individuals with the lowest health expense, while the price is 17% lower (i.e., $9.5) for agents with the highest health expense.
It is also worth noting that the impact on the demand for annuities is quantitatively small. In the model, most agents still choose to annuitize a major portion of their wealth, which is not the case in the data. For instance, Lockwood (2013) finds that the average annuity ownership rate is only 6% in the Health and Retirement Study (HRS) dataset. This suggests that the correlation between health expenses and survival probabilities may not be an important reason why so many Americans do not buy annuities, and there must exist other explanations that are more important for accounting for the non-annuitization in the data.16

Fig. 2 plots the life cycles of consumption and savings for an average agent in the models with and without annuities (note that here the savings include both assets). As can be seen, agents in the model with annuities save slightly less than those in the model without annuities before retirement, but after retirement they dissave more slowly and thus after age 75 they hold more savings than in the model without annuities. The intuition behind this is simple. In the model with annuities, as the decreasing survival probability after retirement is offset by the increasing return on annuities, agents tend to dissave more slowly after retirement. However, annuitization also reduces accidental bequests, therefore they receive less bequest transfer during the working age. As a result, agents in the model with annuities save slightly less before retirement. The consumption profiles are also different in the two models. In the model with annuities, the consumption profile does not significantly decreases after retirement, while it declines quickly after retirement in the model without annuities. The intuition for this result is similar to that for the savings profiles. The return on annuities increases as age increases and the survival probability declines. Therefore, simply according to the Euler equation, the consumption path in the model with annuities should not decline as much as in the model without annuities (see Davies, 1981).

5.3. Precautionary saving

Economists have long argued that uncertain health expenses generate precautionary saving. Recently, there has been a growing macro-literature that uses quantitative dynamic models to study the impact of uncertain health expenses on precautionary saving. Most studies in this literature find that the impact of uncertain health expenses on precautionary saving is large and quantitatively important for understanding the saving and wealth data in the US.17 In this section, I ask whether the implication for precautionary saving is different when uncertain health expenses are correlated with uncertain longevity.18

It is worth mentioning that some existing studies also implicitly captured the correlation between health expenses and longevity. For instance, De Nardi et al. (2010) captured the correlation between health expenses and survival probabilities via including health status as a state variable in their model. Kopecky and Koreshkova (2009) partially captured the correlation by modeling a nursing home shock. In this paper, I assume that the survival probability is directly conditioned on current health expense, and measure the magnitude of the correlation between the health expense and survival probability from the MEPS dataset.

To quantitatively assess the implication of the correlation between health expenses and survival probabilities for precautionary saving, I conduct the following counterfactual experiment in the benchmark model: I assume away the correlation between health expenses and survival probabilities by resetting the survival probabilities for everyone at each age to be the average survival probability at that age, and then recompute the decision rules. As shown in Table 9, the average amount of wealth accumulated at the beginning of retirement increases by 3.5% when health expenses are assumed to be independent of survival probabilities. Across the income distribution, the retirement wealth for agents with the highest productivity only increases by 1.9%, while the retirement wealth for agents with the least productivity increases by 11.5%. The intuition for this result is simple: when the uncertain health expenses are correlated with the survival probabilities, the two uncertainties partially offset each other, thus generating less precautionary savings than in the model without the correlation.

5.4. The impact of medicare

It is also interesting to consider the impact of medicare in the model. To quantitatively assess the effects of medicare on the demand for health insurance and individual welfare, I conduct the following counterfactual experiment: I remove the medicare program in the benchmark model with health insurance, and then recompute the individuals’ decision rules and

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16 See Pashchenko (2013) for a comprehensive review of the literature on the annuity puzzle.

17 Hubbard et al. (1995), De Nardi et al. (2010), Kopecky and Koreshkova (2009), etc.

18 Note that the findings here are also related to the papers that study the saving effects of uncertain life span, such as Davies (1981) and Leung (1994).
the equilibrium. As for the welfare analysis, I adopt the equivalent consumption variation (ECV) as the welfare criteria, that is, the change in consumption each period required for a new born to achieve the same expected lifetime utility. The results are reported in Table 10. As can be seen, medicare has a significant crowding out effect on the demand for private health insurance. When the medicare program is removed, the average private health coinsurance rate chosen increases from 66.3% to 77.0%. Medicare also has a significant effect on individual welfare. In terms of the equivalent consumption variation, individual welfare increases by 5.1% when the medicare program is removed. An important reason for this welfare result is that medicare discourages capital accumulation and thus reduces the aggregate output as it is a pay-as-you-go program. That is, when medicare is removed, the output per capita increases by 10.8% (i.e., from $46,743 to $50,476). In addition, as can be seen, the welfare gain is relatively larger for the least productive agents. This is because the least productive agents are more likely to rely on the welfare program when hit by bad health shocks, and thus medicare is less useful for them.

Note that the welfare result obtained here should be treated with caution, because the model does not include all the relevant elements for a complete welfare analysis of medicare and thus may not capture all the potential welfare effects. For instance, the model does not capture the potential frictions in the private health insurance market (such as adverse selection and administrative costs), and thus may underestimate the benefits from the insurance provided by medicare.

5.5. Further discussion

The main finding of the paper is that the correlation between health expenses and survival probabilities can explain why many agents (especially the rich) do not buy more health insurance. However, it is worth mentioning that there also exist other explanations for the lack of health insurance puzzle, such as the existence of the means-tested programs and the supply-side frictions in the health insurance market.\footnote{See Pauly (1990), Cutler and Gruber (1996a, 1996b), Brown and Finkelstein (2007, 2008), Lockwood (2013), etc.} For instance, Brown and Finkelstein (2007, 2008) found that the existence of medicaid is an important reason why individuals in the bottom half of the distribution do not buy extra health insurance, but the supply-side frictions (i.e., insurance premium markups) are quantitatively not very important.
This paper is complementary to these existing studies. As is known from the data, many rich people also do not buy extra private health insurance. As presented in Table 11, the private insurance coverage rates for individuals aged 65+ are low among all education groups in the MEPS dataset. Individuals with no high school diploma are least likely to buy private health insurance, i.e., only 17.8% of them hold private health insurance. While the rest of the individuals are more likely to buy private health insurance than those without a high school diploma, the coverage rates for them are still relatively low, ranging from 32.5% to 44.7%. This paper contributes to the literature providing a complementary explanation that can help explain why many rich individuals also do not want to buy additional private health insurance.

It is important to note that the model studied here also has limitations as it has left out some elements that are relevant and may be important for understanding the demand for health insurance. For instance, De Nardi et al. (2010) documented in the data that individuals with higher permanent income tend to have more health expenses and live longer. These features are not incorporated in the model, which may have led the model to miss certain dimensions of the data, i.e., the relationship between the health insurance coverage rate and the permanent income. Another limitation is that I do not model the spousal effect in this paper. If the extra private health insurance is the long term care insurance, then the informal care potentially available from family members (e.g., the spouse) could also crowd out the demand for private health insurance. In addition, the model does not include bequest motives. As shown in Lockwood (2013), the presence of bequest motives would decrease the demand for private insurance as bequest motives reduce the opportunity cost of precautionary saving. By leaving these channels out, the model may overestimate the demand for private health insurance. In this paper, I leave these elements out of the model because the focus of the paper is on the new mechanism, i.e., the implication of the correlation between health expenses and survival probabilities, but these channels are definitely important for a comprehensive understanding of the demand for health insurance and need more study in the future.

6. Conclusion

This paper studies a dynamic life cycle model with health shocks that can simultaneously increase health expenses and reduce longevity. I show that rational agents would neither fully insure their uncertain health expenses nor fully annuitize their wealth because the correlation between health expenses and longevity provides a self-insurance channel for both uncertainties. That is, when the agent is hit by a health shock (which simultaneously increases health expenses and reduces longevity), she can use the resources originally saved for consumption in the reduced period of life to pay for the increased health expenses. Since the two uncertainties partially offset each other, the precautionary saving generated in the model should be smaller than in a standard model without the correlation between health expenses and longevity. I calibrate the model using the MEPS dataset, and find that the health expenses are highly correlated with the survival probabilities. The quantitative exercises suggest that the correlation between health expenses and survival probabilities significantly reduces the demand for actuarially fair health insurance, while its impact on the demand for annuities and precautionary saving is relatively small.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.euroecorev.2015.01.003.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>No high school diploma</th>
<th>High school graduate</th>
<th>Some college</th>
<th>College graduate</th>
</tr>
</thead>
<tbody>
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<td>Years of schooling</td>
<td>&lt; 12</td>
<td>12–13</td>
<td>14–15</td>
<td>16+</td>
</tr>
<tr>
<td>Coverage rate</td>
<td>17.8%</td>
<td>32.5%</td>
<td>39.2%</td>
<td>44.7%</td>
</tr>
</tbody>
</table>

Data source: MEPS.

20 Here the private insurance coverage rate is defined as the fraction of individuals with private insurance.

21 As shown in Kopecky and Koreshkova (2009) among others, an important type of health expenses facing the elderly but not covered by medicare is long term care expenses. Thus, long term care insurance is one of the most important additional private health insurance types available to the elderly.
References


