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Social Security, Differential Fertility, and the Dynamics of the Earnings Distribution*

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Abstract

Economists and demographers have long argued that fertility differs by income (differential fertility), and that social security creates incentives for people to rear fewer children. Does the effect of social security on fertility differ by income? Does social security further affect the dynamics of the earnings distribution through its differential effects on fertility? We answer these questions in a three-period OLG model with heterogeneous agents and endogenous fertility. We find that given its redistributional property, social security reduces fertility of the poor proportionally more than it reduces fertility of the rich. Assuming that earning ability is transmitted from parents to children, the differential effects of social security on fertility can have a significant impact on the dynamics of the earnings distribution: a relatively lower fertility rate among the poor can lead to a new earnings distribution with a smaller portion of poor people and a higher average earnings level. With reasonable parameter values, our numerical exercise shows that the effects of social security on differential fertility and the dynamics of the earnings distribution are quantitatively important.

KEYWORDS: social security, differential fertility, earnings distribution, growth

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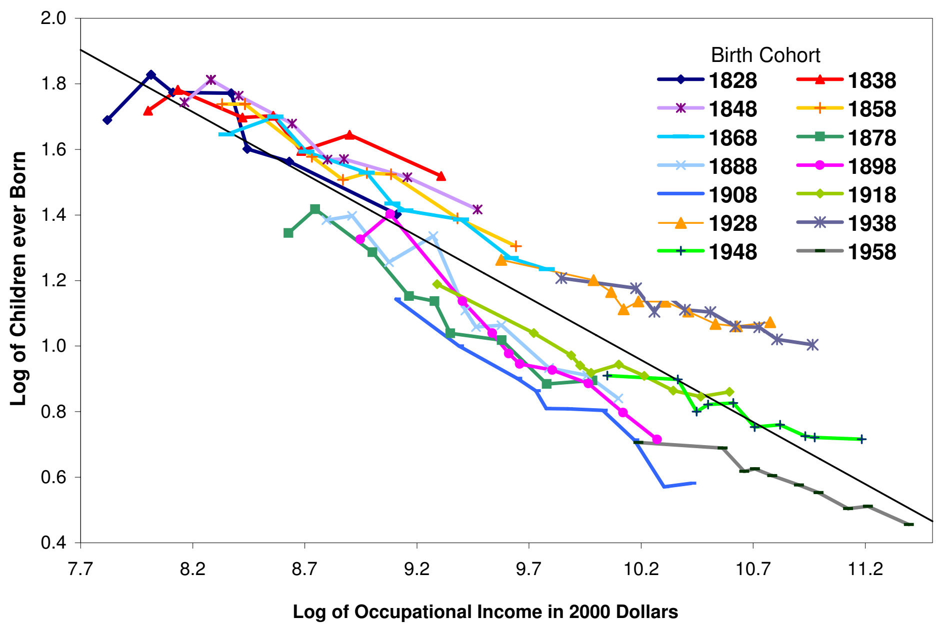
1 Introduction

Fertility differs by income (differential fertility). For example, Jones and Tertilt (2008) find a negative cross-sectional relationship between income and fertility within all cohorts of women since 1826 in the United States (see figure 1). This negative relationship between income and fertility has also been found in most other countries.[1] Economists and demographers have also argued that government-provided social security creates incentives for people to rear fewer children.[2] In this paper, we ask whether the effect of social security on fertility varies across the earnings distribution, and how social security affects the dynamics of the earnings distribution through its differential effects on fertility.

Figure 1: Differential fertility by income in the U.S.: 1826–1960. (from Jones and Tertilt (2008))



To answer these questions, we develop a three-period OLG model with heterogeneous agents and endogenous fertility. In the model, we follow the line of the "old-age security" hypothesis (see Boldrin and Jones (2002) for details) and assume

[1] See Jones (1982).

[2] For example, Nishimura and Zhang (1992), Boldrin, De Nardi, and Jones (2005), Ehrlich and Kim (2007), Bohacek and Belush (2009), etc.

that children are investment goods from the viewpoint of parents. Thus the optimal fertility choice depends on how much transfer is expected from children in relation to the cost of rearing these children to adult life. The cost of rearing children is only the parent's time. We further assume that earning ability is transmitted from parents to children, and it is mean-reverting over generations. Therefore, in this model, poor (rich) parents tend to have more (fewer) children since they have lower (higher) child-rearing cost and expect that their children will have higher (lower) earnings than themselves and give back relatively more (less) in transfers.

In this setup, social security payments crowd children out of parents' old-age portfolios. Since government-provided social security is usually very progressive, its payments are a larger portion of old-age savings for poor people.[3] Therefore, social security tends to reduce the fertility of the poor proportionally more than it reduces the fertility of the rich. Since earning ability is intergenerationally correlated, a smaller fertility differential between the poor and the rich can lead to a new earnings distribution with a smaller portion of poor people and a higher average earnings level.

A crucial assumption in the model is that earning ability is intergenerationally correlated and mean-reverting. These assumptions are supported by overwhelming empirical evidence.[4] Using the intergenerational data from the National Longitudinal Survey, Zimmerman (1992) found that lifetime earnings are strongly correlated between fathers and sons. Solon (1992) found similar results using the intergenerational data from the Panel Study of Income Dynamics. Solon (2002) summarizes empirical studies for a number of developed countries, which all find supportive evidence for the argument that earnings are intergenerationally correlated and mean-reverting.

Another important assumption in our model is that the parent's motivation for child-rearing is old-age security.[5] This is in contrast to the Barro-Becker model,

[3] OECD (2007) summarizes the progressivity of pension systems in the OECD countries. It finds that New Zealand and Ireland have a pure flat-rate pension system, while Finland, Italy and Netherlands have a highly earnings-related pension system. The US social security system is somewhere in between these two groups of countries.

[4] Zimmerman (1992), Solon (1992), and Solon (2002), etc.

[5] One criticism of the "old-age security" hypothesis is that the development of credit markets over the last several decades has weakened the old-age security role of children. Therefore, old-age security should not be the main motive of fertility for parents in modern societies. It is true that the development of credit markets has provided people alternative ways to secure their old age. However, it is fair to argue that children still have many favorable features compared with private savings in terms of old-age security, especially given the fact that the private annuity market is still largely missing. According to Warshawsky (1988), the private annuity coverage was only around 2% among the elderly for most years from 1919 to 1984.

in which parental altruism is the motivation of child-rearing (Barro and Becker (1989), and Becker and Barro (1988)). This "old-age security" hypothesis was first proposed by Caldwell (1978) in the demography literature. Boldrin and Jones (2002) formalize Caldwell's idea in a dynamic model of fertility. Our model builds on the Boldrin-Jones model. Sizable empirical evidence has been found supporting the "old-age security" hypothesis. For instance, Billari and Galasso (2008) use the Italian pension reforms of the 90s as a natural experiment to test the "old-age security" hypothesis. They find that even contemporary fertility in developed countries is consistent with the "old-age security" hypothesis. Boldrin and Jones (2002) provide an excellent review of earlier empirical evidence supporting this hypothesis.[6]

Our paper is motivated by the recent literature that studies the interactions between differential fertility and income and wealth inequality. The key message from this literature is that what really matters is not only the aggregate fertility measures, but also the distribution of fertility across income. For instance, De la Croix and Doepke (2003) argue that differential fertility puts more weight on the poor in the next generation, and thus brings down the weighted average income level of the population. Knowles (1999) argues that accounting for differential fertility is important for understanding the U.S. wealth distribution, since parents treat children's human capital as part of their wealth. Kremer and Chen (2002) argue that differential fertility increases the proportion of unskilled workers, and reduces their wages. Our study builds on this literature and focuses on how government-provided public pension affects differential fertility and its interaction with the dynamics of the earnings distribution.

This paper is closely related to papers by Boldrin and Jones (2002) and Boldrin, De Nardi and Jones (2005). In both papers, children are an investment in parents' old-age consumption and the old-age transfer from children to parents is endogenous. Boldrin and Jones (2002) find that their model can explain the historical correlation between the mortality rate and fertility rate. However, they abstract from distributional issues by only studying a representative-agent model. Boldrin, De Nardi and Jones (2005) study the negative impacts of social security on fertility in the Boldrin-Jones model. They find that social security reduces the period Total Fertility Rate (TFR) and this effect is quantitatively important, but they do not explore the possibility that social security may affect people's fertility behavior differentially. In fact, the literature on social security and fertility can be dated back to the 1960s. Most studies have found that social security has a negative

[6]These earlier studies include Willis(1982), Lillard and Willis (1997), Nugent (1985), and Jensen (1990).

impact on the fertility rate.[7] However, no one has studied the differential effects that social security has on fertility and its implications for the dynamics of the earnings distribution.

This paper also fits into the literature that studies the cross-sectional properties of various family decisions and their interactions with inequality, using a heterogeneous-agent model.[8] Among this literature, there are several papers especially relevant to this paper. Caucutt, Guner and Knowles (2002) argue that both fertility and the timing of fertility differ across the income distribution. They develop an equilibrium search model with marriage and fertility and study the interactions between wage inequality and differential marriage and fertility decisions of young women. Another important family decision which may have a significant impact on fertility is female labor participation. Schoonbroodt (2003) studies the cross-sectional relationship between female labor participation and income in the US. She finds that the increase in the female labor participation rate has been independent of income throughout the twentieth century in the U.S.

The contributions of this paper are twofold: 1) we are the first to explore the possibility that social security may affect the dynamics of the earnings distribution through its differential effects on fertility, and show that this effect is quantitatively important. 2) We propose a novel mechanism generating the negative cross-sectional relationship between fertility and income, in which the mean-reversion in earnings over generations plays a key role. We show that this mechanism is robust both theoretically and quantitatively.

The rest of the paper is organized as follows. We describe the model in the second section and present some theoretical analysis on the cross-sectional relationship between income and fertility in the third section. We illustrate the effect of social security on differential fertility and its implication for the dynamics of the earnings distribution in the fourth section. We conclude in the fifth section.

2 The Model

2.1 The Economic Environment

Consider an economy inhabited by overlapping generations of agents who live for three periods: childhood, middle age, and old age. Agents are endowed with one unit of productive time only in their middle age. They can use it either to work

[7] Friedlander and Silber (1967), Hohm (1975), Swilder (1983), Nishimura and Zhang (1992), Cigno and Rosati (1996), Rosati (1996), Zhang (2001), Zhang, Zhang and Lee (2001), etc.

[8] For example, Caucutt, Guner and Knowles (2002), Greenwood, Guner and Knowles (2003), Schoonbroodt (2003), and Bar and Leukhina (2007).

or to rear children. Agents receive a productivity shock ε at the beginning of the middle-age period and then jointly make savings, fertility, consumption, and old-age transfers decisions to maximize their lifetime utility. In old age, agents only consume what they have, which includes the savings from their middle age and the old-age transfers from their middle-age children. In childhood, agents don't make any economic decision. Let us think about the problem facing a middle-age agent with productivity shock ε_t^i, born in period $t-1$. Here $i \in 1,...,n_{t-1}$ and n_{t-1} is the fertility choice of this agent's parent. This agent has the following expected value of her lifetime utility,

$$u(c_t^m) + \beta E[u(c_{t+1}^o)] + \gamma u(\sum_{j \neq i, j=1}^{n_{t-1}} d_t^j + T_t + d_t^i) \tag{1}$$

with

$$u(c) = \log c, \tag{2}$$

where c^m is middle-age consumption and c^o is old-age consumption. Note that d is the transfer to the old-age parent, and is assumed to be nonnegative ($d \geq 0$). The social security payment to the old-age parent is denoted by T_t. Let j be the index for the middle-age children of the agent's parent, thus d_t^j represents the old-age transfer from the jth child. Here we assume the agent i takes d_t^j, $j \neq i$ and $j = 1,...,n_{t-1}$, as given when she makes her own transfer decision.[9]

The first term and the second term in equation (1) are respectively the utility function for the agent's middle age and old age. The expectation is over the uncertainty a middle-age agent has about her children's productivity and thus her old-age consumption at the moment she makes the fertility choice. The third term in equation (1) is the altruism function, which says that a middle-age child cares about her old-age parent. It is a function of the agent's own transfer to her parent d_t^i, her siblings' total transfers to her parent $\sum_{j \neq i, j=1}^{n_{t-1}} d_t^j$, and the social security payment to her parent T_t. We exclude the parent's own old-age savings from the altruism function in order to prevent the parent from playing strategically with her children.[10] This specification of the altruism function has several convenient fea-

[9] In other words, we assume that middle-age children play a noncooperative game when they make old-age transfer decisions (see Boldrin and Jones (2002)). Note that fertility is a continuous variable in the benchmark model, and a continuous number of children playing a noncooperative game could be a bit problematic. This is a feature of the Boldrin-Jones type model of fertility. Following Boldrin and Jones (2002), we use symmetry for old-age transfers of children when we actually compute the model. In Appendix 2, we show that this problem does not significantly affect our results by studying a version of the model with discrete fertility and heterogenous preference.

[10] The problem would be much more complicated if the parent plays strategically with her children. Corner solutions may yield higher utility for the parent, which means that she may choose

tures. First, the agent's incentive for giving decreases as she has more siblings. Second, the agent's incentive for giving decreases when her parent receives social security transfers. For simplicity the productivity shock is assumed to be the same across all children within a family, which means the productivity shock only refers to the family shock in this paper.[11] Let β be the discount factor and γ be the relative weight on the altruism function. Agents don't derive utility in childhood; they are taken care of by their parents in this period.

The budget constraints for the middle age and the old age are respectively:

$$d_t^i + s_t + c_t^m + bW_t\varepsilon_t^i(1-\tau)n_t = W_t\varepsilon_t^i(1-\tau), \tag{3}$$

$$c_{t+1}^o = R_{t+1}s_t + T_{t+1} + n_t D_{t+1}(n_t, \varepsilon_{t+1}), \tag{4}$$

where $bW_t\varepsilon_t^i(1-\tau)n_t$ is the total cost of rearing children, which means that rearing one child costs a b fraction of the parent's lifetime earnings. A natural interpretation of b is that child-rearing needs parental time, therefore the cost of child-rearing is the forgone earnings.[12] Note that $D_{t+1}(.,.)$ is the children's policy function for old-age transfers in period $t+1$, n is the fertility choice, and s is saving. They satisfy the conditions: $s \geq 0$, and $n \in [0, \widehat{n}]$, where $\widehat{n}$ is the upper limit for fertility choice.[13]

The agent's problem (P1) can be formulated as a Bellman's equation,

$$V(n_{t-1}, \varepsilon_t^i) = \max_{d,n,s} u(c_t^m) + \beta E[u(c_{t+1}^o)] + \gamma u\Big(\sum_{j\neq i, j=1}^{n_{t-1}} d_t^j + T_t + d_t^i\Big)$$

subject to (3) and (4).

either zero savings or zero fertility. We exclude the parent's own old-age savings from the altruism function because it allows us to avoid this complication and also the strategic behavior problem is not the focus of our paper. The strategic behavior problem has been analyzed in some previous studies (O'Connell and Zeldes (1993) and Lagerlof (1997), etc.).

[11] Note that this assumption may overstate the uncertainty on future old-age transfers, because it implies that if the parent gets a unproductive child, all her children would be unproductive, thus the total transfers from children would be very low. The significance of this issue depends on the size of the family and the correlation between children's productivities within a family. Pay-as-you-go social security social security provides insurance against uncertain old-age transfers by pooling contributions from all the middle-age agents and equally redistributing back to the old-age agents. Since our model overstates the uncertainty on future old-age transfers, we expect that the insurance role of social security in our model may also be larger than otherwise it would be.

[12] It is worth mentioning that the child-rearing cost in this model can also be interpreted as goods cost, as long as the goods cost is positively related to the parent's lifetime earnings. Since rich parents usually feed their children with better food and provide them bigger houses, it is reasonable to assume that the goods cost of child-rearing is positively related to the parent's lifetime earnings.

[13] Here $\widehat{n}$ can be understood as the physical limit, which satisfies $\widehat{n} \leq 1/b$.

The productivity shock $\varepsilon_t \in \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_m\}$, is governed by a Markov chain with transition matrix $\pi(i,j) = Prob(\varepsilon_{t+1} = \varepsilon_j | \varepsilon_t = \varepsilon_i)$. The Markov chain is approximated from the log-normal AR(1) process

$$ln\varepsilon_{t+1} = \rho ln\varepsilon_t + u_{t+1}, u_{t+1} \sim N(0, \sigma_u^2), \forall t \tag{5}$$

where ρ is the intergenerational persistence of productivity, and $\rho \in (0,1)$.

Production is undertaken in a firm in accordance with

$$Y_t = K_t^{\alpha}(AL_t)^{1-\alpha}, \tag{6}$$

where K_t is aggregate capital in period t, and L_t is aggregate labor in period t. Let A denote the labor-augmented productivity factor, which is assumed to be constant. In other words, there is no technological progress in the benchmark model. Let $\alpha \in (0,1)$, and let capital depreciate at a rate of δ. The firm chooses inputs by maximizing profits, $Y_t - W_t L_t - (R_t - 1 + \delta)K_t$.

Denote the distribution of the middle-age generation by a density function $\phi_t(n_{t-1}, \varepsilon_t)$. Then the aggregate population of the middle-aged generation, N_t, evolves over time according to:

$$N_{t+1} = N_t \sum_{i=1}^{m} \int_0^{\widehat{n}} \phi_t(n, \varepsilon_i) G_t(n, \varepsilon_i) dn, \tag{7}$$

where $G(.,.)$ is the middle-age agent's policy function for fertility in period t. Here population growth can be easily derived from equation (7): $\widetilde{n}_t = \sum_{i=1}^{m} \int_0^{\widehat{n}} \phi_t(n, \varepsilon_i)$ $G_t(n, \varepsilon_i) dn$.

The density function $\phi_t(n_{t-1}, \varepsilon_t)$ evolves according to:

$$\phi_{t+1}(n_t, \varepsilon_{t+1} = \varepsilon_i) = \frac{\pi(j,i) \int_0^{\widehat{n}} \phi_t(n, \varepsilon_j) G_t(n, \varepsilon_j) I(G_t(n, \varepsilon_j) = n_t) dn}{\sum_{j=1}^{m} \int_0^{\widehat{n}} \phi_t(n, \varepsilon_j) G_t(n, \varepsilon_j) dn}. \tag{8}$$

Here, I(.) is the indicator function.

Denote the policy function for saving by $H(.,.)$, the market clearing conditions for capital and labor are:

$$K_{t+1} = N_t \sum_{i=1}^{m} \int_0^{\widehat{n}} \phi_t(n, \varepsilon_i) H_t(n, \varepsilon_i) dn, \tag{9}$$

and:

$$L_{t+1} = N_{t+1} \sum_{i=1}^{m} \int_0^{\widehat{n}} \phi_{t+1}(n, \varepsilon_i) \varepsilon_i (1 - bG_{t+1}(n, \varepsilon_i)) dn. \tag{10}$$

The social security system is characterized by a payroll tax rate τ_t, and a social security payment T_t. Note that in each period t, $\{\tau_t, T_t\}$ should satisfy the following budget constraint of the government,

$$N_{t-1}\sum_{i=1}^{m}\int_0^{\hat{n}}\phi_{t-1}(n,\varepsilon_i)T_t dn = W_t L_t \tau_t. \tag{11}$$

The left-hand side is the total social security payments in period t. The right-hand side is the total social security revenue in period t.

Definition 1: *a **competitive equilibrium:** Given an initial distribution of a middle-age generation $\phi_0(n_{-1},\varepsilon_0)$, an initial stock of physical capital K_0, and an initial population of the middle-age generation N_0, a competitive equilibrium consists of sequences of prices $\{W_t, R_t\}$, government parameters $\{\tau_t, T_t\}$, aggregate quantities $\{L_t, K_{t+1}, N_{t+1}\}$, distributions $\phi_{t+1}(n_t,\varepsilon_{t+1})$ and policy functions $\{D_t(.,.), G_t(.,.), H_t(.,.)\}$ such that:*

1. *the policy functions $\{D(.,.), G(.,.), H(.,.)\}$ solve the agent's problem (P1);*

2. *the firm's choices L_t and K_t maximize profits;*

3. *the prices W_t and R_t are such that markets clear, i.e. conditions (9) and (10) are satisfied;*

4. *the distribution evolves according to (8) ; and population, N_t, evolves according to (7); and*

5. *the government budget constraint* (11) *is satisfied.*

Definition 2: *a **stationary equilibrium** is a competitive equilibrium where the density function, $\phi(.,.)$, prices, $\{R,W\}$, the policy function for the old-age transfer, $D(.,.)$, the population growth rate, $\widetilde{n}$, and social security parameters, $\{\tau, T\}$, are all constant over time.*

In the rest of the paper, we focus on stationary equilibria.

3 Key Forces Underlying Differential Fertility

In this section, we analytically show why the poor choose to have more children than the rich in the model. The first order conditions (FOC) for the individual's problem are the following,

$$u'(c_t^m) = \gamma u'(\sum_{j\neq i, j=1}^{n_{t-1}} d_t^j + T_t + d_t^i), \tag{12}$$

$$u'(c_t^m) = \beta R_{t+1} E[u(c_{t+1}^o)], \tag{13}$$

$$u'(c_t^m) b W_t \varepsilon_t^i (1-\tau) = \beta E[u'(c_{t+1}^o)(D_{t+1} + n_t \frac{dD_{t+1}}{dn_t})], \tag{14}$$

where $u'()$ represents the first order derivative. Equation (12) is the FOC for the transfer choice, d. The left-hand side is the marginal cost of the transfers, and the right-hand side is the marginal benefit of the transfers. The FOC clearly implies that agents reduce their transfers when their parents receive more transfers from other sources, such as their siblings or the social security program. Agents also reduce their transfers when they have lower earnings. Equation (13) is the FOC for the saving choice, which is a standard Euler equation. Equation (14) is the FOC for the fertility choice. The left-hand side of the equation is the marginal loss of having children. The right-hand side is the marginal benefit of having children. As can be seen, the uncertainty surrounding children's productivity makes it difficult for us to see how people's earnings affect their fertility choices from the FOCs. In order to illustrate the key forces underlying the fertility differential between the poor and the rich in the model, we switch to analyze a simplified version of the model in the rest of this section.

The simplified version is primarily different in two ways. First, we assume that within each family i, the intergenerational productivity process follows the linear equation $\varepsilon_{t+1}^i = g^i \varepsilon_t^i$ instead of equation (5). Here g^i measures how fast the productivity grows over generations within family i, and it is assumed that $g^i > 0$.[14] Second, it is assumed that prices, R and W, are exogenously given and constant over time. In other words, this is a partial equilibrium model. We also abstract from social security and assume away the altruism weight $(\gamma = 1)$. Thus the middle-age agent's problem for family i is the following:

$$\max_{d,n,s} u(c_t^m) + \beta u(c_{t+1}^o) + u(\sum_{j \neq i, j=1}^{n_{t-1}} d_t^j + d_t^i) \tag{15}$$

subject to

$$c_t^m = W\varepsilon_t^i - s_t - d_t - W\varepsilon_t^i b n_t \tag{16}$$

$$c_{t+1}^o = Rs_t + D_{t+1}(n_t) n_t. \tag{17}$$

Here $D_{t+1}(n_t)$ is children's decision rule for old-age transfers, which is a function of n_t. It is easy to see that in a steady state it must be that $n_t = n_{t+1}$, $s_{t+1} = g^i s_t$, and

[14] Note that the linear equation $\varepsilon_{t+1}^i = g^i \varepsilon_t^i$ is not a special case of equation (5), and g does not correspond to ρ in equation (5). However, the linear equation does capture the key features of the intergenerational productivity process in the full-blown model. That is, productivity is intergenerationally correlated and mean-reverting (assuming poorer families have a higher value of g).

$d_{t+1} = g^i d_t$ for each family i. Proposition 1 highlights the mechanism underlying differential fertility in this model.

Proposition 1: *Suppose that there are two families i and j, with productivity ε_t^i and ε_t^j respectively. They face intergenerational persistence of productivity, g^i and g^j, so that $\varepsilon_{t+1}^i = g^i \varepsilon_t^i$ and $\varepsilon_{t+1}^j = g^j \varepsilon_t^j$, for all t. Then the following two statements are true:*

1. *If $g^i = g^j$, then in the steady state the two families have the same fertility rate, $n^i = n^j$, regardless of their productivity levels.*
2. *If $g^j < g^i < R(1+\beta)$, then in the steady state family i has a higher fertility rate than family j, $n^i > n^j$, regardless of their productivity levels.*

Proof: in appendix 1.

The key message of this proposition is that the intergenerational productivity process is all that matters for the family's fertility choice. The first statement implies that a poor family has the same fertility rate as a rich family if its productivity grows at the same rate over generations as the rich family. The second statement says that under a certain condition ($g < R(1+\beta)$), a family facing a steeper upward intergenerational productivity process has more children, regardless of the productivity levels. The two statements together imply that if poorer families have a higher value of g (mean-reverting productivity), then poorer families would have a higher fertility rate, and they do so not because they are poorer, but because they are expecting their productivity is growing faster over generations. The intuition behind Proposition 1 is simple. Since in this model children are an investment (alternative to saving) for the parent's old-age security, the optimal fertility choice depends on the return from investing in children relative to the return from saving. A steeper upward intergenerational productivity process implies a relatively higher return from investing in children and therefore causes the family to have more children to substitute for saving in the old-age portfolio.[15]

Note that the positive relationship between n and g holds only when the value of g is under a certain level ($g < R(1+\beta)$). The intuition for this is the following. As g keeps rising, the parent's saving will eventually reach the corner

[15]Proposition 1 tells us that the mean reversion of earnings over generations can generate a negative cross-sectional relationship between fertility and earnings. This is not only true in the "old-age security" type model of fertility. Zhao (2009) shows that the mean reversion of earnings over generations can also generate a negative relationship between fertility and earnings in the Barro-Becker type model of fertility.

solution (go down to zero). After that, the rise in g has two opposite effects on the fertility choice. On the one hand, it increases the return from investing in children thus having a positive effect on the fertility choice. On the other hand, given constant fertility, the rise in g increases the total amount of old-age transfers that the parent receives, thus it lowers the marginal benefit of an extra child to the parent and has a negative effect on the fertility choice. The net effect of the rise in g on the fertility choice relies on the relative importance of these two effects.

4 Differential Effects of Social Security

In this section, we first demonstrate that the negative relationship between fertility and earnings remains in the full-blown model (in the case of no social security). We then illustrate the effect of social security on differential fertility and its further implications for the dynamics of the earnings distribution. We rely on numerical methods. Specifically, we answer the following two questions:

1. How does the cross-sectional relationship between fertility and earnings change as the size of social security increases?

2. How does the earnings distribution change as the size of social security increases?

4.1 Parameterization

We start by parameterizing the model. There are in total 8 model parameters (see table 1). Some of them are standard parameters in macroeconomic models of fertility, hence, we determine their values based on the existing literature. As for the rest of the parameters, we choose their values to match some empirical moments. We also do extensive sensitivity analysis with respect to a number of key model parameters.

A model period is set to 20 years. We follow the existing literature to determine the values of the standard parameters. That is, we set the value of α to 0.33, the value of β to 0.99 (annual), and the value of δ to 8% (annual). The values of γ and A are normalized to one. The values of ε and its transition matrix are obtained from the following exercise: discretizing the AR(1) process and converting it to nine-state Markov chains according to Tauchen (1986).

Given these choices, we still need to choose the values of the three parameters: ρ, b, σ_μ. In order to pin down them at the same time, we need three empirical moments. Here we choose the intergenerational correlation in lifetime earnings, the childbearing cost, and the Gini of lifetime earnings.

Table 1: Benchmark model Parameterization

Parameter	Value	Description
A	1.0	Labor-augmented productivity factor
γ	1.0	Altruism weight
α	0.33	Capital share
δ	0.08 (annual)	Capital depreciation rate
β	0.99 (annual)	Time discount factor
		Moments to match
b	0.03	Cost of a child (as a fraction of lifetime earnings)
ρ	0.667	Intergenerational correlation coefficient
σ_μ	0.45	Gini of lifetime earnings

The intergenerational persistence of productivity (ability) is measured by ρ in this model. It is important because it directly affects the cross-sectional relationship between fertility and earnings (differential fertility). Using the intergenerational data from the National Longitudinal Survey, Zimmerman (1992) estimated that the intergenerational correlation in lifetime earnings is 0.667. We set the value of ρ to match Zimmermann's estimate. We refer to Sensitivity Analysis for the cases with different values of ρ. According to Boldrin, De Nardi and Jones (2005), the time cost of rearing a child is roughly 3% of the parent's time available for work over the whole working life, which is used to pin down the value of b. Again, we consider different values of b in Sensitivity Analysis. Bowlus and Robin (2004) reported that the Gini coefficient of lifetime earnings is approximately 0.31 in the developed world.[16] We choose the value of σ_μ so that the Gini of lifetime earnings in the model matches the one reported in Bowlus and Robin (2004).

The model matches all three empirical moments very well. All the parameter values are summarized in table 1. This set of parameter values implies a total fertility rate of 2.74, a yearly interest rate of 5.9%, and a capital-output ratio of 2.38 in the benchmark economy (when $\tau = 0$: no social security). We also find that the negative cross-sectional relationship between fertility and earnings remains in the complete model in the case of no social security (see the $\tau = 0$ line in figure 3).

[16] See Bowlus and Robin (2004) for a detailed description of the empirical literature on lifetime earnings inequality.

4.2 The Aggregate Effects of Social Security

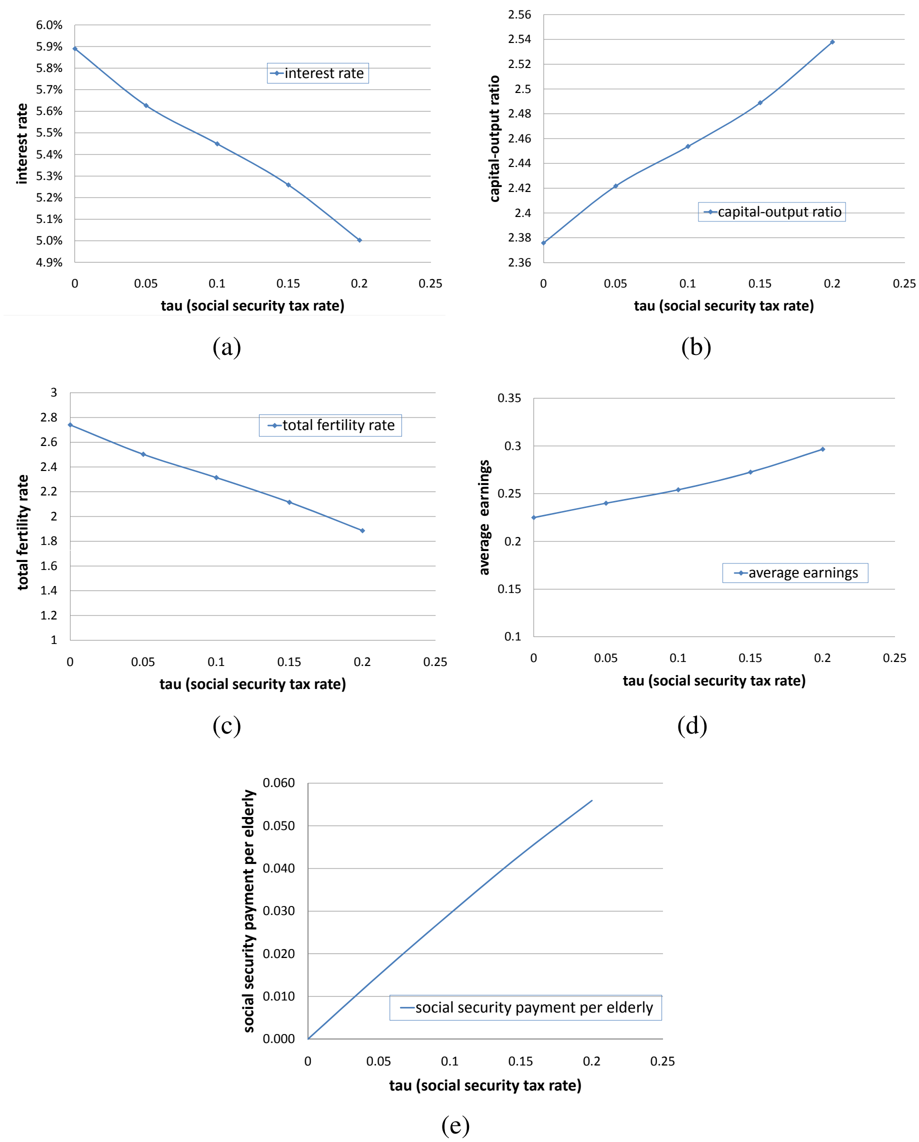
To understand the effects of social security, we simply compare stationary equilibria with different levels of social security (more specifically, different levels of the social security tax rate τ). Figures 2 plots the relationships between τ and several aggregate variables: the interest rate, the capital-output ratio, the TFR, the average earnings, and social security payment per elderly. Figure 2(a) and 2(b) show that when τ increases from 0 to 20%, the interest rate (yearly) slightly decreases from 5.9% to 5.0%, and the capital-output ratio increases from 2.38 to 2.54. Figure 2(c) and 2(d) show that the TFR drops 0.85 from 2.74 to 1.89 and the average earnings increase by 35% from 0.22 to 0.30. Figure 2(e) shows that social security payment per elderly is positively correlated with the value of τ.[17]

As our model is extended from the Boldrin-Jones model, it is natural to compare the aggregate effects of social security in our model to those in the standard Boldrin-Jones model (studied in Boldrin, De Nardi and Jones (2005)). We find that our results confirm what have been found in Boldrin, De Nardi and Jones (2005). The effects of social security on most aggregate variables in our model are consistent with theirs. In particular, we find that as the social security tax rate rises from 0% to 20%, the TFR drops by 0.85 while they found that the TFR drops by 0.76 as the social security tax rate increases from 1% to 20%. This suggests that the standard representative-agent Boldrin-Jones model can well describe the aggregate dynamics of a heterogenous population.

One important thing worth mentioning here is that, in contrast to the previous literature on social security (see Martin Feldstein 1974), the interest rate decreases as the size of the social security program increases in the Boldrin-Jones type model. The intuition for this is the following. The traditional wisdom says the social security payments crowd out the private life-cycle savings, and increase the interest rate. However, in the Boldrin-Jones type model, the social security payments reduce not only the private life-cycle savings, but also the fertility rate which is directly related to the labor supply in the next period. Thus the net effect of social security on interest rate may be negative. Furthermore, as pointed out by Boldrin, De Nardi and Jones (2005), the change in interest rate has a feedback effect on fertility, that is, as the interest rate decreases parents tend to have more children to substitute for saving in their old-age portfolio.

[17] Note that theoretically this may not always hold true because the rise in the value of τ has two opposite effects on social security payment per elderly: while it increases social security revenue per middle-age worker, it also increases the population dependency ratio (more elderly per middle-age worker). Since social security payment per elderly and the value of τ are highly correlated, the negative effect of τ should be quantitatively small in this model.

Figure 2: Aggregate variables and the SS tax



While this effect is economically significant in Boldrin, De Nardi and Jones (2005), it is quantitatively much smaller in our model. The reason for that is, social security has two opposite effects on aggregate labor supply in our model. On the one hand, it lowers the aggregate labor supply by reducing the TFR. On the other hand, it increases the aggregate labor supply by increasing the average productivity level through its differential effect on fertility by income. As a result, the net effect on aggregate labor supply is smaller in our model than in the standard Boldrin-Jones model and thus the decrease in interest rate is also smaller.

4.3 Differential Fertility

Figure 3 plots the fertility-productivity relationships for different levels of social security. It demonstrates that the fertility of those at the low end of the distribution declines much more than other people when the social security tax rate increases. The difference in fertility change makes the fertility-productivity curve flatter when τ increases. This point can be best seen in Table 2, which summarizes the fertility changes of agents with different levels of earning shock as τ increases from 0 to 20%. We can see that fertility declines the most (by 63.0%) for the agents with the lowest productivity, and the drop in fertility gets smaller as ε increases. Fertility only drops by 5.6% for the agent with the highest productivity.

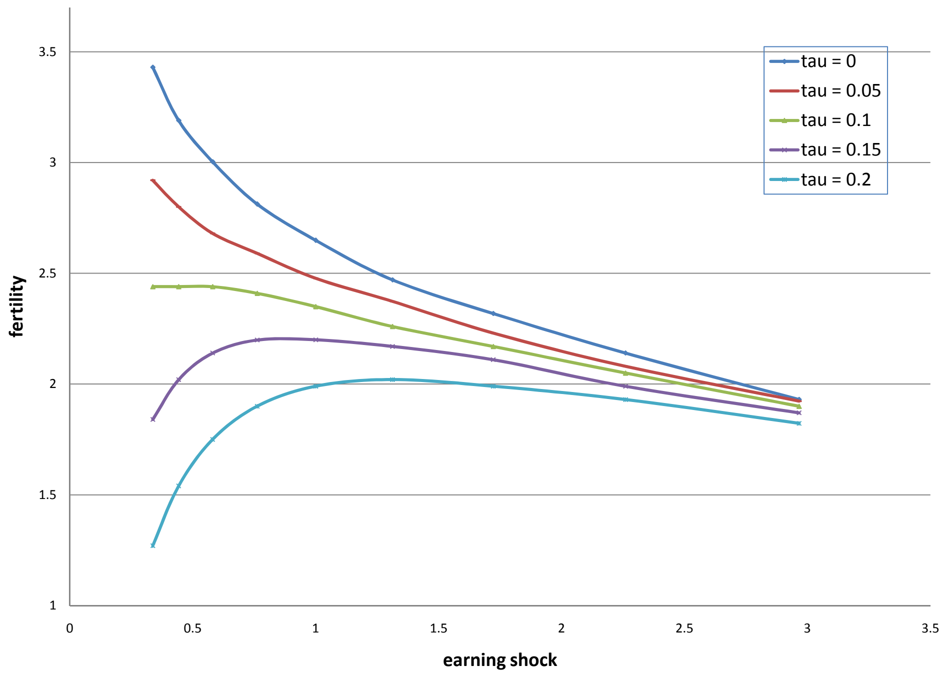
Table 2: Fertility changes by earning shock ε (τ: $0 \rightarrow 20\%$)

Earning shock ($\varepsilon_1 < \varepsilon_2$)	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7	ε_8	ε_9
Fertility drop (%)	63.0%	51.7%	41.7%	32.4%	24.9%	18.2%	14.2%	9.8%	5.6%

4.4 Dynamics of the Earnings Distribution

When taking into account differential fertility, social security affects the earnings distribution through two channels. First, social security changes the composition of the population in the stationary equilibrium by reducing the fertility differential between the rich and the poor. This compositional effect can be best observed by looking at the earnings distributions corresponding to different values of social security tax rate τ (see figure 4). As can be seen, the distribution shifts to the right when the social security tax rate τ increases. The density of poor people decreases as the density of rich people increases. In other words, there are fewer poor people in the population as the size of social security expands. The intuition behind this compositional change is as follows. Since earnings are correlated over generations,

Figure 3: Differential fertility and the SS tax



the poor tend to have children who are also relatively poor. Therefore, when fertility of the poor drops proportionally more than the rich, the portion of poor people in the whole population goes down.

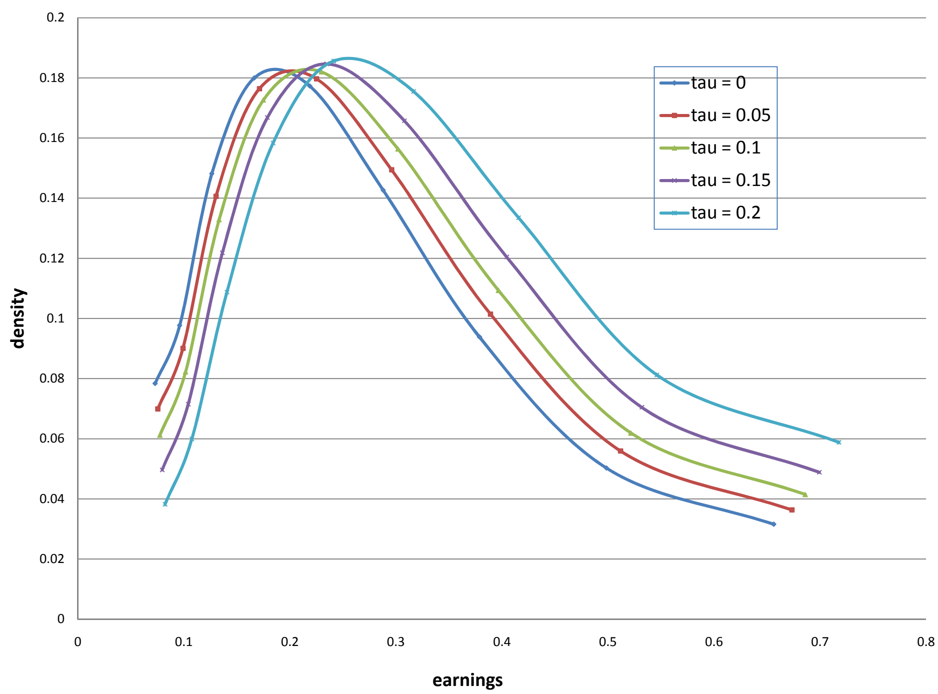
The second channel is that social security increases people's work time by reducing their time devoted to child-rearing. Due to social security's differential effects on fertility, the work time of poor people increases more than that of the rich, and so do their earnings. Table 3 shows this effect clearly. When τ increases from 0 to 20%, the earnings of the least productive people increase by 3.42%, and the earnings of the most productive people increase 0.17%.

We can see that social security does not have significant impacts on the earnings distribution through the second channel, and its main effect on the earnings distribution is the compositional effect via changing differential fertility.

Table 3: Earnings changes by earning shock ε (τ: $0 \to 20\%$)

Earning shock ($\varepsilon_1 < \varepsilon_2$)	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7	ε_8	ε_9
Earnings increase (%)	3.42%	2.60%	1.97%	1.43%	1.03%	0.70%	0.51%	0.33%	0.17%

Figure 4: Earnings distribution and the SS tax



The changes in the earnings distribution lead to corresponding changes in the average earnings of the economy. Since there are fewer low income people as τ rises, the average earnings increase as the social security tax τ increases (see figure 2(d)).[18]

Since we abstract from technological progress, there is no economic growth in stationary equilibria in this model. However, an increase in the size of social security can generate sizable economic growth on the transition path between two stationary equilibria. We can see in figure 2(d), when the social security tax rate increases from 0% to 10%, the average earnings increase by 13%. A further increase from 10% to 20 % will lead to a 17% increase in average earnings. Furthermore, since one model period corresponds to one generation, the growth on the transition path may last over decades.

[18]Note that part of the increase in average earnings is also due to the small increases in wage and labor supply.

4.5 Poverty Reduction

One interesting implication of the analysis conducted above is that social security can reduce poverty by changing the composition of the population.[19] By lowering the relative fertility rate of the poor, social security in the long run reduces the share of population trapped in poverty. This can be best seen in Table 4, which presents the poverty rates in stationary equilibria with different social security tax rates. The poverty measure adopted in the table is the poverty headcount ratio, which is one of the most widely-used poverty measures, and the poverty line is set to 50% of the population mean income. Therefore, the poverty rate here means the percentage of population whose income are below 50% of the population mean income.

Table 4: Poverty rates by τ: $0 \to 20\%$

Social security tax rate	$\tau = 0$	$\tau = 5\%$	$\tau = 10\%$	$\tau = 15\%$	$\tau = 20\%$
Poverty rate (%)	17.1%	15.9%	13.8%	11.7%	9.9%

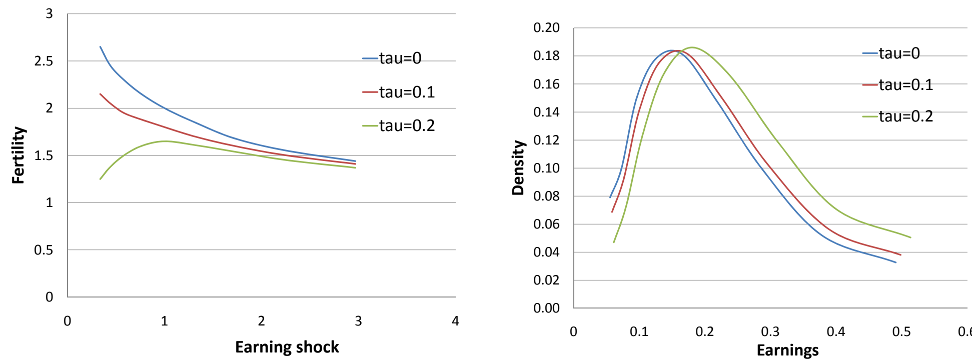
As we can see in Table 4, the poverty rate is 17.1% when there is no social security, and it drops to 9.9% when the social security tax rate increases to 20%, which suggests that social security can play a significant role in poverty reduction.

4.6 Sensitivity Analysis

In this section, we do sensitivity analysis with respect to a number of key parameters, β, b, and ρ. The main conclusion from the sensitivity analysis is: the qualitative predictions of the model do not change when the values of these parameters vary. As shown in figure 5(a)-10(a), the fertility of those at the low end of the distribution declines much more than other people when the social security tax rate increases. As a result, the fertility-productivity curve becomes flatter as the value of τ increases. The dynamics of the earnings distribution is also similar with that in the benchmark case. As can be seen in figure 5(b)-10(b), the distribution shifts to the right as the value of τ increases. That means, there are fewer poor people in the population as the size of social security expands. Note that the quantitative implications of the model do change as some key parameter vary, i.e. the intergenerational

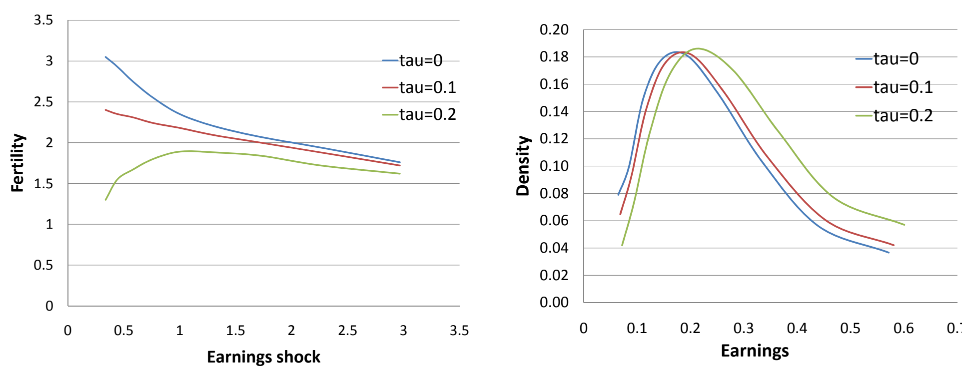
[19]This is a new mechanism. The existing literature argues that the main mechanism through which social security reduces poverty is as following: social security improves the living standard of the existing people (mainly the elderly) (see Atkinson (1989)). This paper shows that social security may also reduce poverty by changing the composition of people with different productivity levels (fewer people with low productivity).

Figure 5: Sensitivity analysis: $\beta = 0.95$



(a) (b)

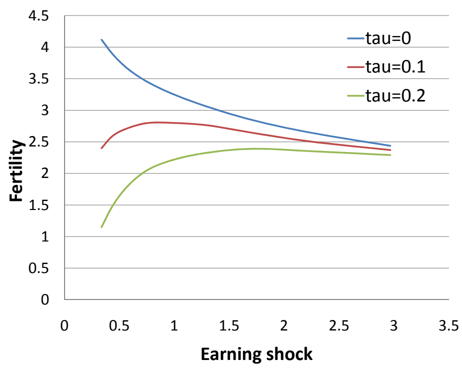
Figure 6: Sensitivity analysis: $\beta = 0.97$



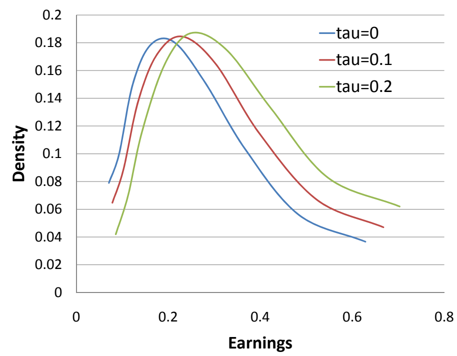
(a) (b)

persistence of productivity, ρ. The model predictions are quantitatively more important as the value of ρ increases. The intuition behind is the following. A high value of ρ means that poor parents tend to have children who are still very poor and rich parents tend to have children still very rich. As a result, the substitution effect of lump-sum social security payments on fertility is large for the poor and small for the rich. This implies that social security has a large effect on differential fertility and the earnings distribution.

Figure 7: Sensitivity analysis: b=0.02



(a)

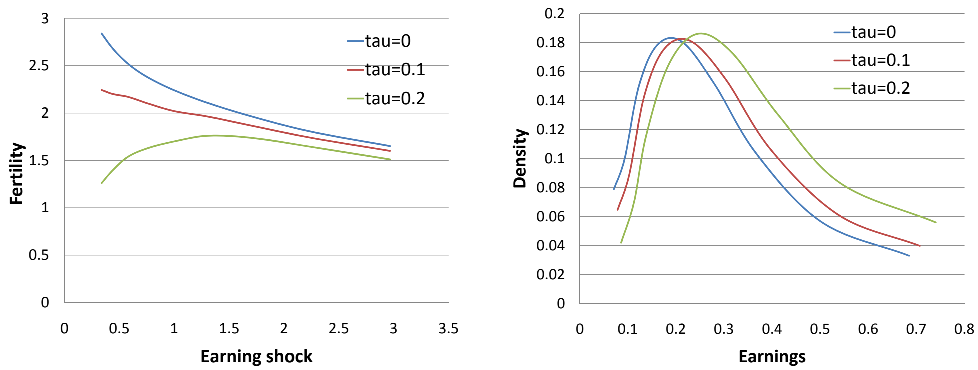


(b)

5 Conclusion

In this paper, we study the effects that government-provided social security has on differential fertility (by income) and its implications for the dynamics of the earnings distribution. As shown in previous literature, social security has negative effects on fertility when children are treated as parents' old-age security. We find that given its redistributional property, social security reduces the fertility of the poor proportionally more than that of the rich, and therefore reduces the fertility differential between the poor and the rich. We further show that this reduction in differential fertility generates a new stationary distribution with a smaller proportion of poor people and raises the economy's average earnings. It is easy to see that in this model, rearing children and saving are two alternative ways to secure people's old age. Hence, fertility should have a significant impact on saving. We find that in the benchmark model the poor have a lower saving rate than the rich (as shown in figure 11). The reason for that is the following. In a Boldrin-Jones type model of fertility, the poor rely more on their children's transfers after retirement, therefore they tend to save less for their old age. A similar result is found in Scholz and Seshadri (2009), who study the effects of children on household wealth accumulation in a Barro-Becker model of fertility. It is worth mentioning that the underlying forces driving this result may be very different in these two fertility models. Children are treated as consumption goods in the Barro-Becker model of fertility while in a Boldrin-Jones type model they are investment goods. It will be interesting to study the quantitative effect of children on wealth accumulation and the related policy implications in the Boldrin-Jones type model of fertility and com-

Figure 8: Sensitivity analysis: b=0.04



(a) (b)

pare them with what have been found in the Barro-Becker model of fertility (e.g. Scholz and Seshadri (2009)).[20] We leave this for future research.

Appendix 1: proof of proposition 1

At the steady state, the FOCs of the middle-age agent's problem are:

$$\frac{1}{\sum_{j\neq i,j=1}^{n_{t-1}} d_t^j + d_t^i} = \frac{1}{c_t^m} \tag{18}$$

$$R\beta \frac{1}{c_{t+1}^o} = \frac{1}{c_t^m} \tag{19}$$

$$\frac{\partial c_{t+1}^o}{\partial n_t} \beta \frac{1}{c_{t+1}^o} = \frac{W\varepsilon_t^i b}{c_t^m} \tag{20}$$

Imposing symmetry in siblings' transfer decisions, and rearranging the FOCs above gives:

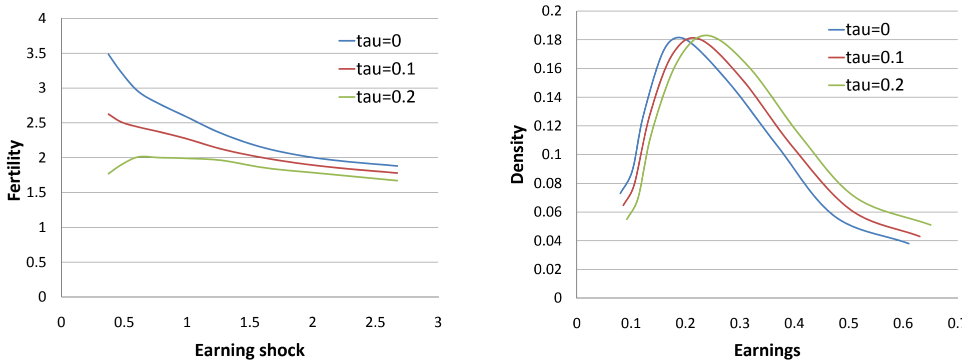
$$n_{t-1} d_t = c_t^m \tag{21}$$

$$R\beta c_t^m = c_{t+1}^o \tag{22}$$

$$\frac{\partial c_{t+1}^o}{\partial n_t} \beta c_t^m = W\varepsilon_t^i b c_{t+1}^o \tag{23}$$

[20] To do so, we need to extend the model to include more realistic elements that are relevant with saving behavior, such as survival probabilities, income shock over the working life, credit constraint, and accidental bequests.

Figure 9: Sensitivity analysis: $\rho = 0.567$



(a) (b)

Substituting in the budget constraint (16) and solving equation (21) for d_t gives:

$$d_t = \frac{1}{1+n_{t-1}}(W\varepsilon_t^i - s_t - W\varepsilon_t^i b n_t) \tag{24}$$

Substituting this into the old age budget constraint gives:

$$c_t^o = Rs_{t-1} + \frac{n_{t-1}}{1+n_{t-1}}(W\varepsilon_t^i - s_t - W\varepsilon_t^i b n_t) \tag{25}$$

After some algebra, we obtain the rate of return on children:

$$\frac{\partial c_{t+1}^o}{\partial n_t} = \frac{1}{(1+n_t)^2}(W\varepsilon_{t+1}^i - s_{t+1} - W\varepsilon_{t+1}^i b n_{t+1}) \tag{26}$$

Remember that at steady state we have the following conditions: $n_t = n_{t+1}$, $s_{t+1} = g^i s_t$, $\varepsilon_{t+1} = g^i \varepsilon_t$, and $d_{t+1} = g^i d_t$ for each family i. Dropping the time index, substituting in the rate of return on children, and combining equations (22)and (23) gives:

$$W\varepsilon^i bR = g^i \frac{1}{(1+n^i)^2}(W\varepsilon^i - s^i - W\varepsilon^i b n^i) \tag{27}$$

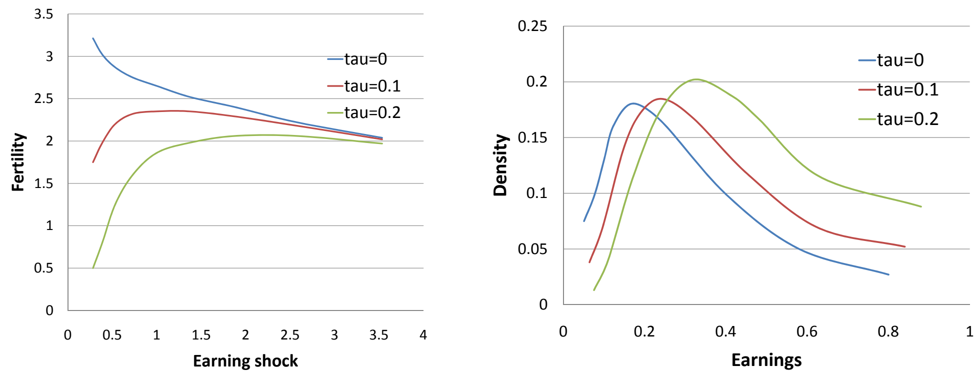
Combining equations (21) and (22), and some algebra gives:

$$s^i = \frac{\frac{n^i}{n^i+1}(W\varepsilon^i - W\varepsilon^i b n^i)(\beta R - g^i)}{R + \frac{n^i}{n^i+1}(\beta R - g^i)} \tag{28}$$

By substituting (28) into (27), we cancel out s^i and get the following equation:

$$(R^2 + R(\beta R - g^i))(n^i)^2 + (2R^2 + R^2\beta)n^i + R^2 - \frac{g^i R}{b} = 0 \tag{29}$$

Figure 10: Sensitivity analysis: $\rho = 0.767$



(a) (b)

Equation (29) is the equation determining the fertility choice of the family i. We can see equation (29) does not contain ε^i. The only family-specific parameter it contains is g^i. Thus it is obvious that, for any two families i and j with $g^i = g^j$, we have $n^i = n^j$, regardless of the levels of ε^i and ε^j. The first statement is proved.

The second statement says if $R + R\beta > g^i$ is satisfied, n^i increases as g^i increases. To prove it, we first need to derive $\frac{\partial n^i}{\partial g^i}$. We get $\frac{\partial n^i}{\partial g^i}$ by applying implicit function theorem to equation (29).

$$\frac{\partial n^i}{\partial g^i} = -\frac{\frac{\partial F(,)}{\partial g^i}}{\frac{\partial F(,)}{\partial n^i}} = \frac{R(n^i)^2 + \frac{R}{b}}{2(R^2 + R(\beta R - g^i))n^i + 2R^2 + R^2\beta} \tag{30}$$

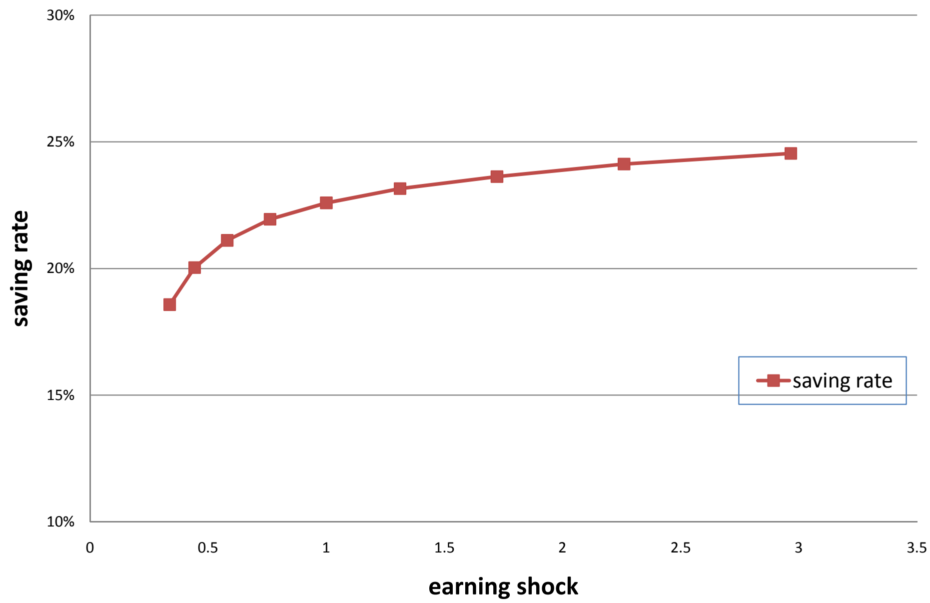
It can be easily seen that, $\frac{\partial n^i}{\partial g^i} > 0$, if $R^2 + R(\beta R - g^i) > 0$, or $R + R\beta > g^i$. Q.E.D.

Appendix 2: discrete fertility choice

As we said in footnote 9, assuming continuous fertility may cause a measurement problem. We take a shortcut by using symmetry for old-age transfers of children when we actually solve the model. In order to show that this problem does not significantly affect our results, we study a version of the model with discrete fertility choice and heterogenous preference. In this version of the model, agents also differ by altruism toward their parents (γ). The middle-age agent's problem can be written as follows:

$$V(\varepsilon_{t-1}, n_{t-1}, \varepsilon_t^i, \gamma_t^i) = \max_{d,n,s} u(c_t^m) + \beta E[u(c_{t+1}^o)] + \gamma_t^i u\Big(\sum_{j\neq i, j=1}^{n_{t-1}} d_t^j + T_t(\varepsilon_{t-1}) + d_t^i\Big)$$

Figure 11: Saving rate by productivity in the benchmark economy (no social security)



subject to

$$d_t^i + s_t + c_t^m = W_t \varepsilon_t^i (1-\tau)(1-bn_t), \tag{31}$$

$$c_{t+1}^o = R_{t+1} s_t + T_{t+1}(\varepsilon_t^i) + n_t D_{t+1}(\varepsilon_t^i, n_t, \varepsilon_{t+1}, \gamma_{t+1}). \tag{32}$$

Here the altruism weight $\gamma_t \in \{\gamma_1, \gamma_2, ..., \gamma_m\}$, is governed by a Markov chain with transition matrix $\pi_\gamma(i, j) = Prob(\gamma_{t+1} = \gamma_j | \gamma_t = \gamma_i)$. The Markov chain is approximated from the AR(1) process

$$\gamma_{t+1} = (1-\rho_\gamma)\mu + \rho_\gamma \gamma_t + \nu_{t+1}, \nu_{t+1} \sim N(0, \sigma_\nu^2), \forall t, \tag{33}$$

where μ is the unconditional mean of γ, ν is the standard deviation, and ρ is the time persistence coefficient. We set μ to one, ν to 0.2, and ρ_γ to 0.667, and compute this version of the model (the rest of parameter values are the same as those in the benchmark parameterization). Figure 12 and 13 show the analogs of the results presented in figure 3 and 4. We find that this version of the model produces very similar results as the benchmark model does. This means that assuming continuous fertility does not significantly affect our results.

Figure 12: Differential fertility and the SS tax (discrete fertility choice)

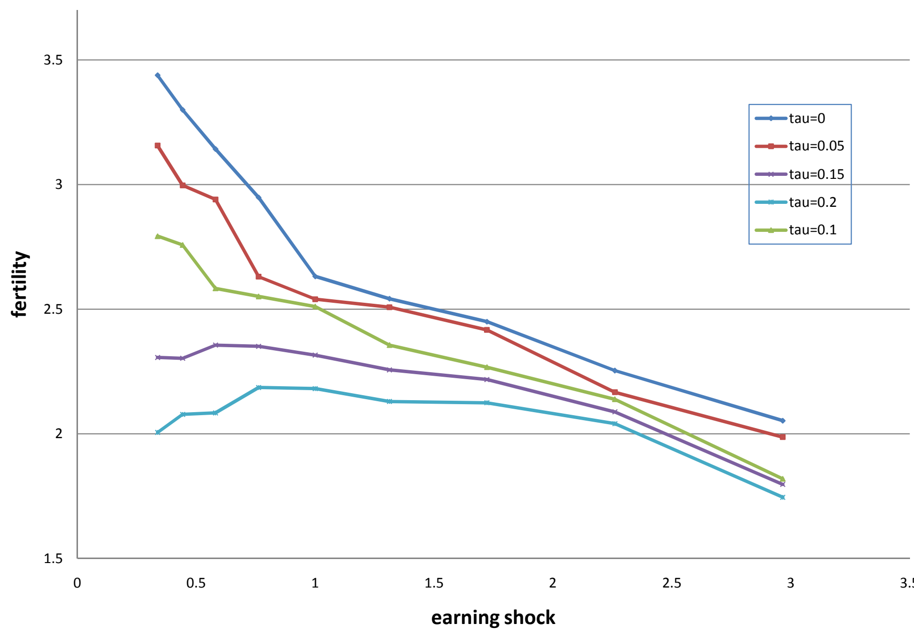
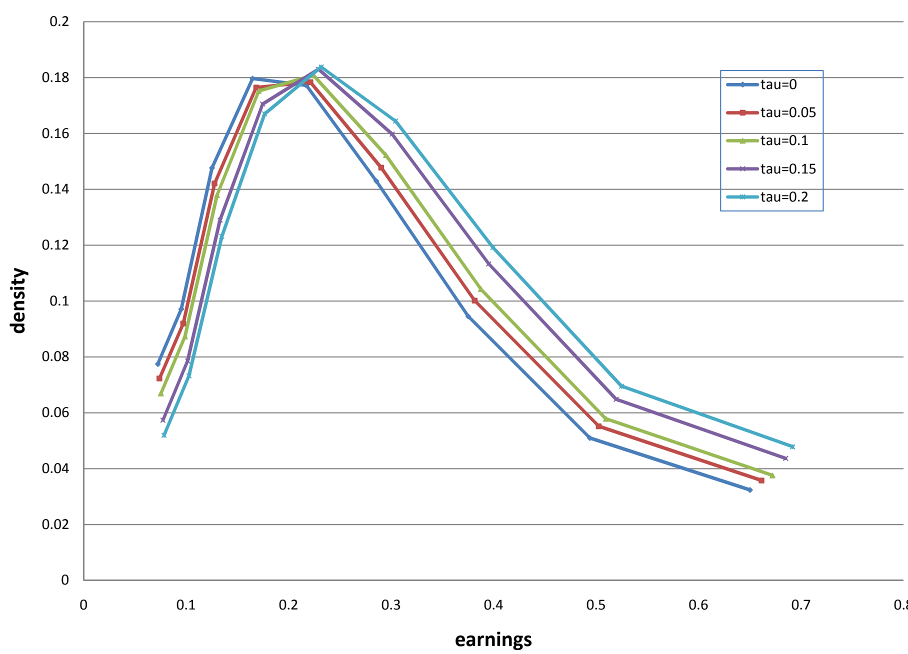


Figure 13: Earnings distribution and the SS tax (discrete fertility choice)



References

Atkinson, A. B. (1989) “Poverty and social security,” London: Harvester Wheatsheaf.

Bar, M., and O. Leukhina (2007) “Accounting for Changes in Labor Force Participation of Married Women: The Case of the U.S. since 1959,” University of North Carolina, working manuscript.

Barro, R. J., and G. S. Becker (1989) “Fertility Choice in a Model of Economic Growth,” *Econometrica*, 57, 481–501.

Becker, G. (1988) “Family Economics and Macro Behavior,” *American Economic Review*, 78, 1–13.

Becker, G., and G. Lewis (1973) “On the Interaction between the Quantity and Quality of Children,” *Journal of Political Economy*, 81(2), S279–88.

Becker, G., and N. Tomes (1976) “Child Endowments and the Quantity and Quality of Children,” *Journal of Political Economy*, 84(4), 143–62.

———. (1986) “Human Capital and the Rise and Fall of Families,” *Journal of Labor Economics*, 4(3), 1–39.

Becker, G. S., and R. J. Barro (1988) “A Reformulatin of the Theory of Fertility,” *Quarterly Journal of Economics*, 103, 1–25.

Bental, B. (1989) “The Old Age Security Hypothesis and Optimal Population Growth,” *Journal of Population Economics*, 1, 285–301.

Billari, F. C., and V. Galasso (2008) “What Explains Fertility? Evidence from Italian Pension Reforms,” *CEPR Discussion Papers* 7014.

Bohacek, R., and V. Belush (2009) “Social Security Reform in a Dynastic Life-Cycle Model with Endogenous Fertility,” unpublished manucript, Economics Institute, Czech Academy of Sciences.

Boldrin, M., M. De Nardi, and L. E. Jones (2005) “Fertility and Social Security,” NBER working paper, 11146.

Boldrin, M., and L. E. Jones (2002) “Mortality, Fertility and Saving in Malthusian Economy,” *Review of Economic Dynamics*, 5, 775–814.

Bowlus, A., and J.-M. Robin (2004) “Twenty Years of Rising Inequality in US Lifetime Labor Values,” *Review of Economic Studies*, 73(1).

Caldwell, J. C. (1978) “A Theory of Fertility: from High Plateau to Destabilization,” *Population and Development Review*, 4, 553–577.

Castaneda, A., J. Diaz-Gimenez, and J.-V. Rios-Rull (2003) "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 111(4), 818–857.

Caucutt, E., N. Guner, and J. Knowles (2002) "Why do Women Wait? Matching, Wage Inequality, and the Incentives for Fertility Delay," *Review of Economic Dynamics*, 5(4).

Cigno, A. (1991) "Economics of the Family," Oxford University Press: Oxford and New York.

Cigno, A., and F. C. Rosati (1996) "Jointly determined saving and fertility behavior: theory and estimates for Germany, Italy, UK, and USA," *European Economic Review*, 40, 1561–1589.

De la Croix, D., and M. Doepke (2003) "Inequality and Growth: Why Differential Fertility Matters," *American Economic Review*, 93(4), 1091–1113.

———. (2004) "Public versus Private Education when Differential Fertility Matters," *Journal of Development Economy*, 73(2), 607–629.

De Nardi, M. (2004) "Wealth Inequality and Intergenerational Links," *Review of Economic Studies*, 71, 743–768.

Echevarria, C., and A. Merlo (1999) "Gender Differences in Education in a Dynamic Household Bargaining Model," *International Economic Review*, 40, 265–286.

Erlich, I., and J. Kim (2007) "Social Security, Demographic Trends and Economic Growth: Theory and Evidence from the International Experience," *Review of Economic Dynamics*, 10(1), 55–77.

Feldstein, M. (1974) "Social Security, Induced Retirement, and Aggregate Capital Accumulation," *Journal of Political Economy*, 82, 905–926.

Friedlander, S., and M. Silver (1967) "A Quantitative Study of the Determinants of Fertility Behavior," *Demography*, 4, 30–70.

Greenwood, J., N. Guner, and J. A. Knowles (2003) "More on Marriage, Fertility, and the Distribution of Income," *International Economic Review*, 44(3), 827–862.

Hugget, M., and G. Ventura (1999) "On the Distributional Effects of Social Security Reform," *Review of Economic Dynamics*, 2(3), 498–531.

Jensen, E. R. (1990) "An Econometric Analysis of the Old Age Security Motive for Child-bearing," *International Economic Review*, 31, 953–968.

Jones, E. (1982) "Socio-economic Differentials in Achieved Fertility," *World Fertility Survey Comparative Study No. 21*, ECE Analyses, Voorburg, The Netherlands: Internatioanal Statistical Institute.

Jones, L. E., and M. Tertilt (2008) "An Economic History of Fertility in the U.S.: 1826-1960," in *Frontiers of Family Economics*, Elsevier, edited by Peter Rupert.

Juster, T., and F. Stafford (1991) "The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement," *Journal of Economic Literature*, 29, 471–522.

Knowles, J. (1999) "Can Parental Decisions Explain U.S. Income Inequality?," Working Paper, University of Pennsylvania.

Kremer, M., and D. Chen (2002) "Income Distribution Dynamics with Endogenous Fertility," *Journal of Economic Growth*.

Lagerlof, N. (1997) "Intergenerational Transfers and Altruism," PhD dissertation, Institute for International Economics Studies, Stockholm University.

Lillard, L., and R. Willis (1997) "Motives for Intergenerational Transfers: Evidence from Malaysia," *Demography*, 34, 115–134.

Maddison, A. (1995a) "Monitoring the World Economy," OECD.

———. (1995b) "Standardized Estimates of Fixed Capital Stock: A Six Country Comparison," *Explaining the Economic Performance of Nations*, Edward Elgar, Aldershot, 137–66.

Moe, K. (1998) "Fertility, Time Use, and Economic Development," *Review of Economic Dynamics*, 1, 699–718.

Nishimura, K., and J. Zhang (1992) "Pay-As-You-Go Public Pensions with Endogenous Fertility," *Journal of Public Economics*, 48, 239–258.

Nugent, J. B. (1985) "The Old Age Security Motive for Fertility," *Population and Development Review*, 11, 75–98.

O'Connell, S. A., and S. P. Zeldes (1993) "Dynamic efficiency in the gifts economy," *Journal of Monetary Economics*, 31(3), 363–379.

OECD (2007) "Pensions at a Glance," Paris, 44–45.

Rosati, F. C. (1996) "Social Security in a Non-Altruistic Model with Uncertainty and Endogenous Fertility," *Journal of Public Economics*, 60, 283–94.

Scholz, J. K., and A. Seshadri (2009) "Children and Household Wealth," University of Wisconsin, mimeo.

Schoonbroodt, A. (2003) “Cross-sectional Properties of Female Labor Force Participation: Theory and Data,” University of Minnesota, mimeo.

Solon, G. (1992) “Intergenerational Income Mobility in the United States,” *American Economic Review*, 82(3), 393–408.

———. (2002) “Cross-Country Differences in Intergenerational Earnings Mobility,” *Journal of Economic Perspectives*, pp. 59–66.

Tauchen, G. (1986) “Finite State Markov Chain Approximations to Univariate and Vector Autoregressions,” *Economics Letters*, 20, 177–181.

Warshawsky, M. (1988) “Private Annuity Markets in the United Stats: 1919–1984,” *Journal of Risk and Insurance*, 55(3), 518–528.

Willis, R. J. (1982) “The Direction of Intergenerational Transfers and Demographic Transition: The Caldwell Hypothesis Reexamined,” *Population and Development Review*, 8, 207–234.

Zhang, J. (2001) “Long-Run Implications of Social Security Taxation for Growth and Fertility,” *Southern Economic Journal*, 67, 713–24.

Zhang, J., J. Zhang, and R. Lee (2001) “Mortality Decline and Long-Run Growth,” *Journal of Public Economics*, 80, 485–507.

Zhao, K. (2009) “War Debt and the Baby Boom,” University of Western Ontario, unpublished manuscript.

Zimmerman, D. J. (1992) “Regression Toward Mediocrity in Economic Stature,” *American Economic Review*, 82(3), 409–429.